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Calculation of Porosity in Clay Formations with examples and a second form of expression

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Abstract: Another proposed expression is presented for the calculation of porosity for reservoirs preferably with a high volume of clay, using sonic tools, mentioning the equations developed over the years and their use in practice, 3 cases will be tested where it will be put into In practice, the equations obtained to calculate the porosity are mentioned as part of the methodology by Karter H. Makar and Mostafa H Kamel (2011), the statistical estimator of quadratic error is considered to check the results.

Keywords: Porosity, Porosity Equation, Wyllie Equation, Raymer Hunt Equation, Raiga Equation, Zenteno Equation, Clay Volume

Introduction and Review of the Classic Equations for the Calculation of Porosity

Over time, numerous methods have been developed for the calculation of this parameter, among which the analysis of cores stands out, however, in the absence of said samples, an estimate is made through the use of geophysical records, which was mentioned previously. and unfortunately it is not enough to measure it, a series of calculations are required to help us minimize the errors produced by the volume of clay present.

An example of equations to obtain such a measurement are those proposed by Wyllie (1956) and Raymer (1980), who considered the effect of the matrix and the fluids in clean formations, using the data derived from the sonic log, which the Delta t means that measured by the geophysical record, that of the matrix and the fluid that invades it.

$$\emptyset_s = \frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}} \quad (1)$$

For unconsolidated sands, Tixier (1959) introduced the compaction factor in the Wyllie formula, resulting in:

$$\emptyset_s = \frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}} \times \frac{1}{C_p} \quad (2)$$

Where Cp the compaction factor and is equal to ((Δ_tsh×C))/100 (C is a constant that is normally 1 and in microseconds per foot as units of measurement in transit time).

In 1980, researcher L.L. Raymer introduced a sonic porosity equation to the industry that continues to be used today. The result of transit time and porosity obtained from other records, for which it is possible to approximate with adequate precision in the areas of interest.

$$\Delta_t = \left[\frac{(1-\emptyset_s)^2}{\Delta_{tma}} + \frac{\emptyset_s}{\Delta_{tf}}\right]^{-1} \quad (3)$$

Finally, Raiga-Clemenceau (1988), had a better approach than Wyllie regarding transit time and porosity:

$$\emptyset_s = 1 - \left(\frac{\Delta_{tma}}{\Delta_t}\right)^{1/x} \quad (4)$$

In summary we can put this table where the advantages and limitations of each mentioned equation are given

Table 1. Equations for calculating porosity (Ref. [1])

REFERENCE	EQUATION	OBSERVATIONS
Wyllie et al., 1956 (More detailed introduction)	$\emptyset_s = \frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tms}}$	Very popular
		It works with consolidated sands and carbonates with intergranular porosity.
Tixier et al., 1959		Gives a good correlation between porosity and transit time interval

	$\emptyset_s = \frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tms}} x \frac{1}{C_p}$	Use 55.5 µs/ft for sands, 49 µs/ft for limestone and 43.5 µs/ft for dolomite.
Raymer et al.(More detailed introduction)	$C_p = \frac{\Delta_{tsh} \times C}{100}$	
		Essentially empirical
	$\Delta_t = \left[\frac{(1-\emptyset_s)^2}{\Delta_{tma}} + \frac{\emptyset_s}{\Delta_{tf}}\right]^{-1}$	Assume that the fluid is a liquid, and not a gas.
		Use 54 µs/ft for sands, 49 µs/ft for limestone and 44 µs/ft for dolomite.
Raiga-Clemenceau et. al.,1988	$\emptyset_s = 1 - \left(\frac{\Delta_{tma}}{\Delta_t}\right)^{1/x}$	Does not detect effects on pore fluids.
		x is the exponent related to the nature of the matrix. 1.6 for sand 1.76 for limestone and 2 for dolomite

Porosity estimation

As mentioned above in qualitative well readings, porosity values are always evaluated from core analysis or porosity log analysis (Density, Neutron, and Sonic). By not having the possibility of performing core analysis, combining at least two porosity tools is useful to evaluate it.

Karter and Mostafa (2011) in the article entitled "An approach to minimize errors in the calculation of the effective porosity in reservoirs of clayey nature in view of the Wyllie-Raymer_Raiga relationship" proposed an equation as an alternative for the determination of parameters in the tools sonic, which contemplates the union of the equations of Wyllie, Raymer and Raiga-Clemenceau.

$$\boldsymbol{\emptyset_s^2 + \left(\frac{\Delta_{tma}}{\Delta_{tf}} - 2\right)\emptyset_s + 1 - \left(\frac{\Delta_{tf} - \Delta_t}{\Delta_{tf} - \Delta_{tma}}\right)^x = 0} \qquad \textbf{(5)}$$

In the previous Article the Equation with the Satisfactory Evaluations was proposed for use

Proposed Equation Z

$$\boldsymbol{\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + 1 - \left[\frac{\Delta tf - \frac{\Delta t}{Cp} + \left(\frac{1-Cp}{Cp}\right)\Delta tma}{\Delta tf - \Delta tma}\right]^x = 0} \qquad \textbf{(6)}$$

According to the exponent x, the new final expression is obtained, if you can see if there is not a significant clay transit time present, Cp = 1 and thus the expression is the same as that proposed by Karter and Mostafa (2011). Without loss of generality when Cp = 1, it is the previous proposed equation. So if we handle the previous Equation more to put it in a simpler sense, we have the following

$$\boldsymbol{\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + 1 - \left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]^x = 0}$$

Now a remark regarding the independent term in Raymer

$$[(1-\emptyset)^x] = \frac{\Delta tma}{\Delta t}$$

$$\left[\left(1 - \left[1 - \left(\frac{\Delta tma}{\Delta t}\right)\right]\right)^x\right] = \frac{\Delta tma}{\Delta t}$$

Let's see this approximation

$$\left[\left(1 - \left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]\right)^x\right] = \left[\left(\left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]\right)^x\right]$$

Let this term be

Of a = $\left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)$therefore it would have an expression of this form

$$[(1 - [1 - a])^x] = 1 - \left[1 + xa + \frac{x(x-1)a^2}{2!} + \cdots\right]$$

With the second approximation truncating us gives us, I have equaled to 0

$$[(1 - [1 - a])^x] = \left[-xa - \frac{x(x-1)a^2}{2!}\right]$$

$$\left[xa + \frac{x(x-1)a^2}{2!}\right] = 0$$

Now rearranging terms, squaring, we get to this

$$1 - a = \left(\frac{x^2}{4} - \frac{1}{4}\right) a^2$$

Completing this expression gives us

$$1 + 2(1 - a) + (1 - a)^2 = -a^2x^2 - 2$$

Where the terms of

$$(1 - a)^2 = -a^2x^2 - 5 - 2a$$

If in the term a on the right hand side is $\left(\frac{\Delta tl - \Delta tma}{Cp*(\Delta tf - \Delta tma)}\right)$ and also setting the expression equal to 0 we can dispense with the right term and thus gives us

$$(1 - a)^2 = -\left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)^2 x^2 - 2\left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right) - 5$$

$$(1 - a)^2 = 0$$

Finally we have

$$[(1 - [1 - a])^x] = (1 - a)^2$$

Where the exponent 2 would be the approximation that x gives for the rock matrix and that same error would be from that same error approximation evaluation 2

$$[(1 - [1 - a])^x] = (1 - a)^x$$

So the approximate expression is

$$\boldsymbol{\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + \left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]^x = 0}$$

Observing how it is approximately, it will be possible that this approximates the porosity even more

$$\left[\left(1 - \frac{\emptyset}{Cp}\right)^x\right] = \frac{\Delta tma}{\Delta t}$$

Table 1

Proposed Equation Z **ECZ1**	$\boldsymbol{\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + 1 - \left[\frac{\Delta tf - \frac{\Delta t}{Cp} + \left(\frac{1-Cp}{Cp}\right)\Delta tma}{\Delta tf - \Delta tma}\right]^x = 0}$ Without loss of generality when Cp=1, is the previous equation proposed
With the Clay Volume Correction.	$\boldsymbol{\emptyset t = \emptyset - Vsh\left[\frac{\Delta tsh - \Delta tma}{\Delta tf - \Delta tma}\right]}$
Proposed Equation Z2	$\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + \left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]^x = 0$
Full Proposal Equation Z2 ECZ2	$\boldsymbol{\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + 1 - \left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]^x = 0}$

Adjustment Indicator

The indicators of deviation of a group of data in relation to a model can be used to assess the goodness of fit between both. Among the most common indicators are the following: RMSE, MAE, NRMSE, CV-MRSE, SDR, and R^2. The one used to determine the degree of error was the Root Mean Square Error (RMSE). Table 2 gives the equations for the fit indicators that have been used by Lu (2003) and Junninen et al. (2002).

Table 2. Adjustment Indicator

Indicator	Equation
Root Mean Square Error (Raíz Cuadrada del Error)	$\boldsymbol{RMSE = \sqrt{\left(\frac{1}{N-1}\right)\sum_{i=1}^{N}(Pi - Oi)^2}}$

Results with the following Wells.

Well 1

Matrix Exponent	Fluid Transit Time	Matrix Transit Time
1.9556	189	43.6
DTsh = 110 us/ft		**Full Proposal Equation Z2 ECZ2**

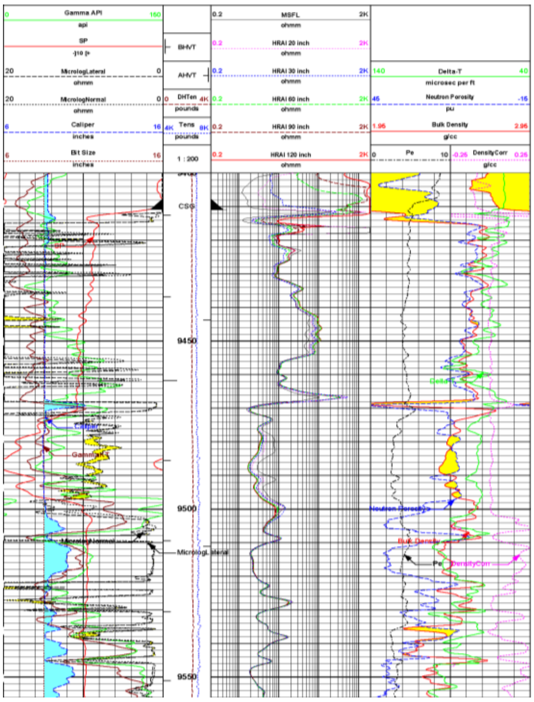
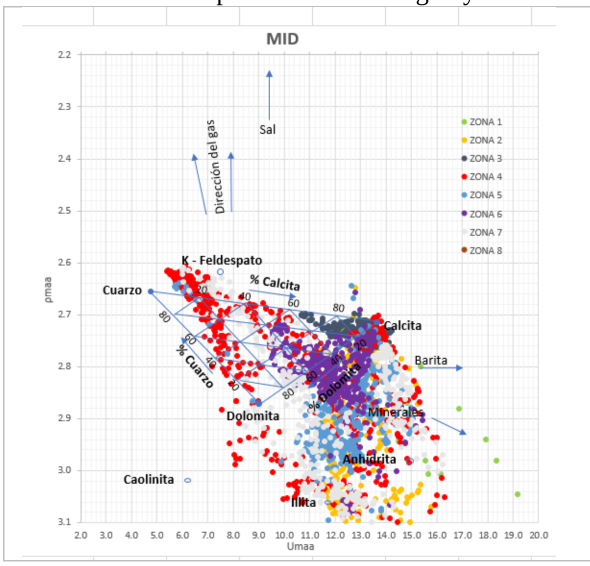
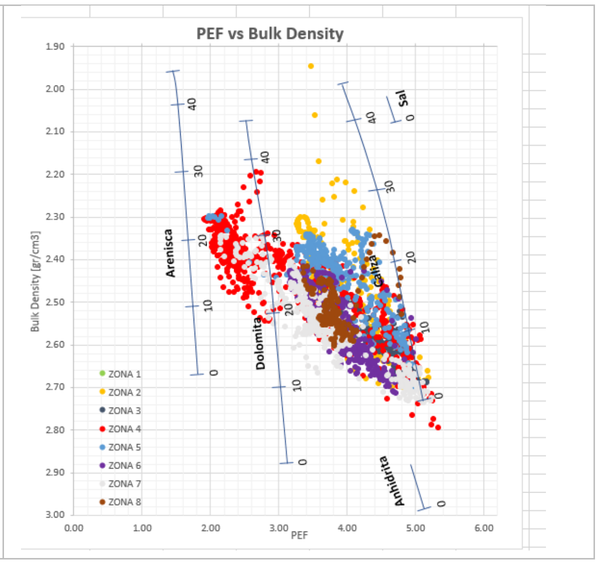


Figure.1 Well Log 1

The Previous Interpretation of the Registry shown below





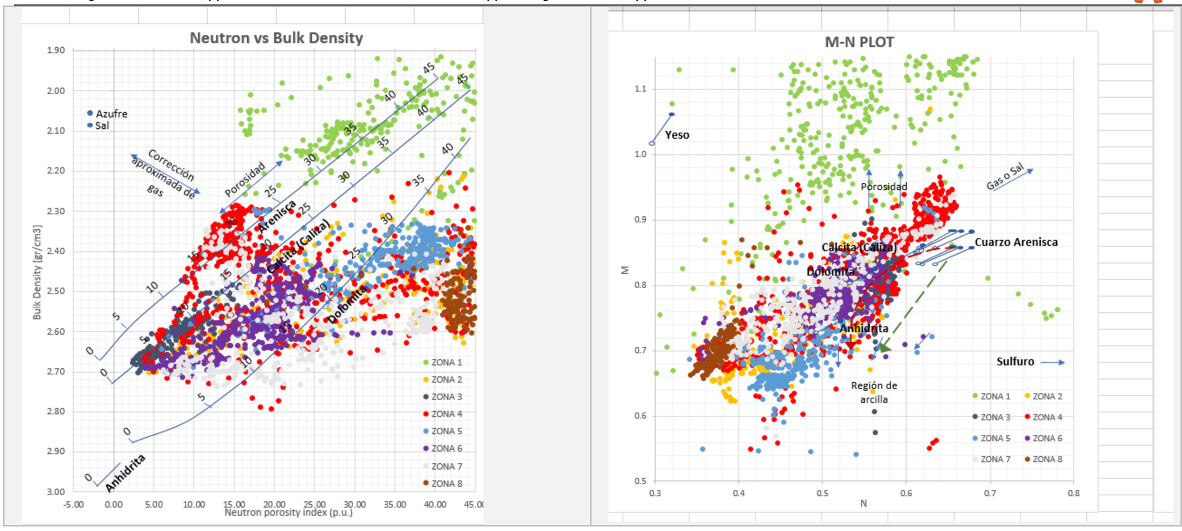


Figure.2Well Log 1 Interpretation.

Records Plotted in Matlab with Results to the Right

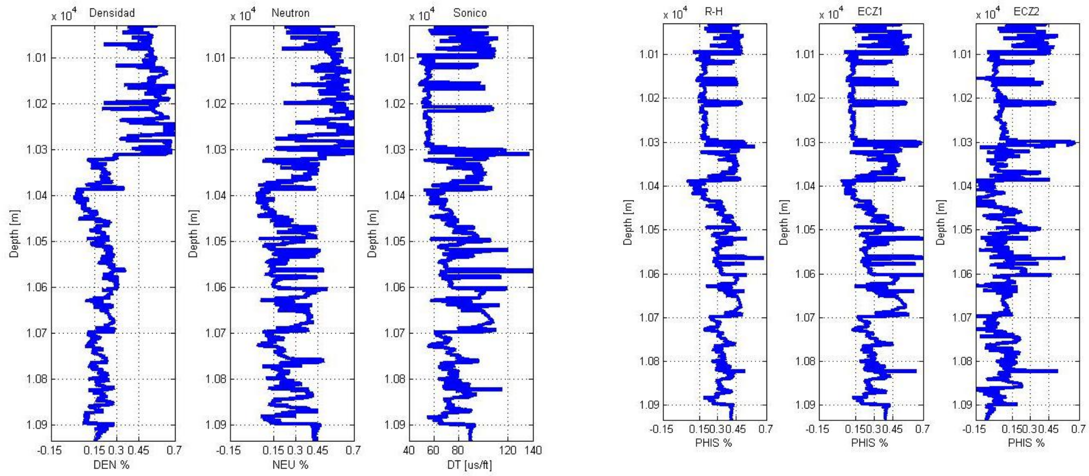


Figure.3Well 1 records and their evaluations.

Porosity Results

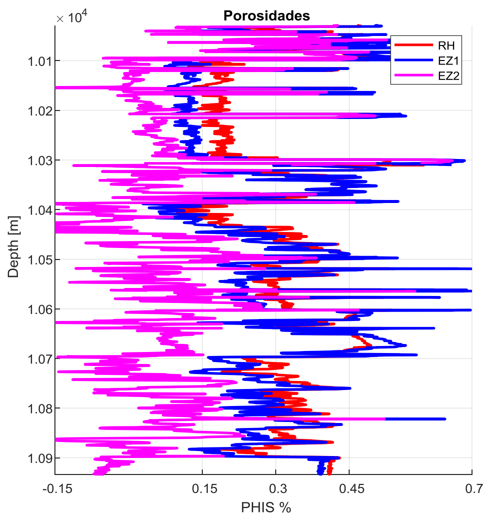
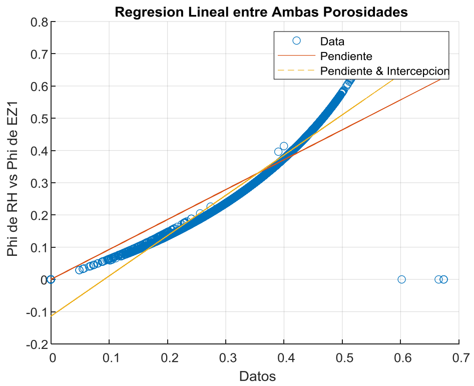


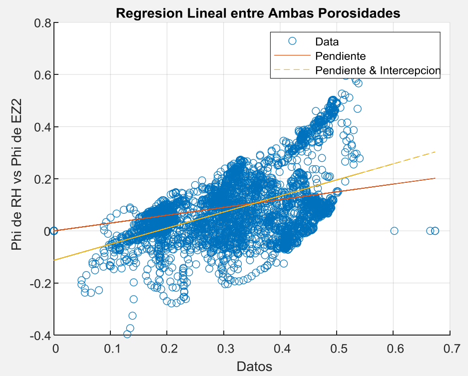
Figure.4Logs from Well 1 and their porosities.

Let's see your Correlations between the Results of the EZ1 and RH

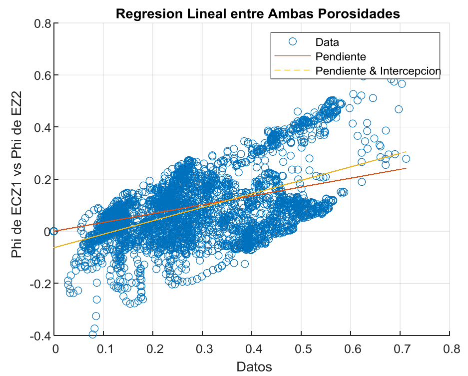
Table 3



There is a good correlation between the two. RMSE between RH as observed and EZ1 as predicted
RMSE = 0.0585

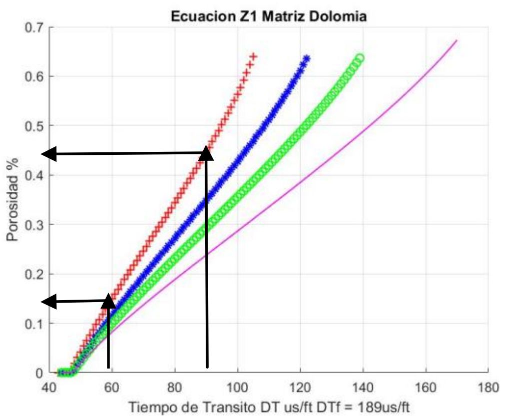


EZ2 and RH
RMSE = 0.2652 Between EZ2 and RH
There is no relationship or even close to the results



EZ1 and EZ2
RMSE = 0.2393

One way to plot Equation Z1



Example at 10040 m DT is 60 us/ft with DTsh = 110 us/ft so approx 14%

The line in red is from DTsh = 110
The blue line is from DTsh=130
The green line is from DTsh=150
The line in pink is from DTsh=180

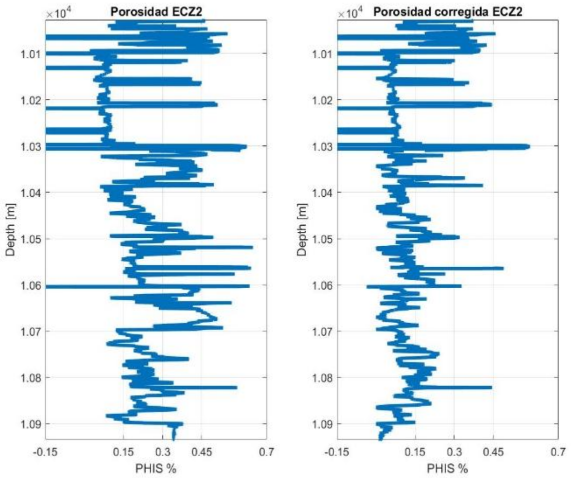
Example at 10070 m DT is 60 us/ft at 90 with DTsh = 110 us/ft thus approx 43%

Now leaving the term -1 in Expression 2, the ECZ2 or the Full Expression.

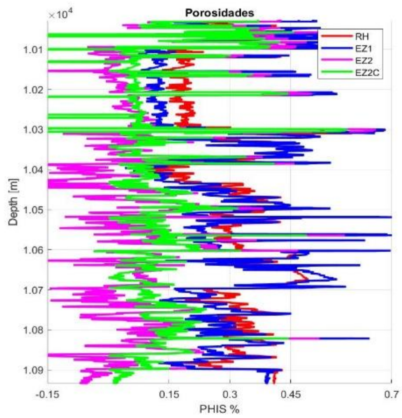
$$\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + 1 - \left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]^x = 0$$

Is obtained

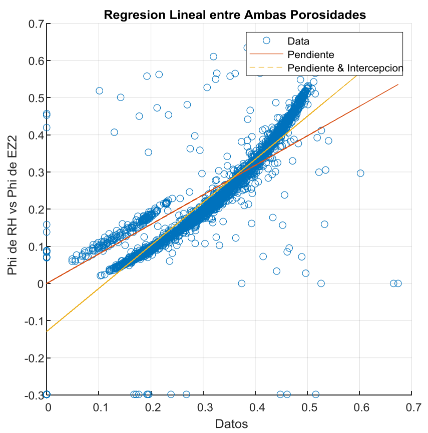
Table 5



With Vsh fixes

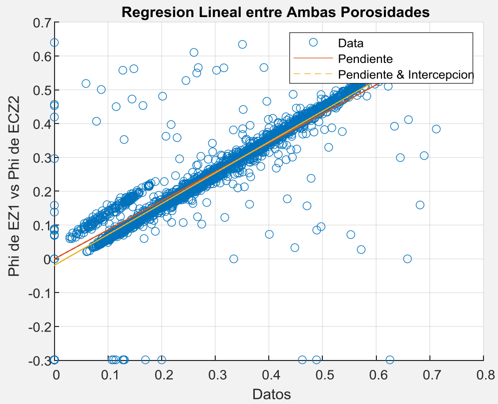


Errors between RH and ECZ2 Complete Table 5



It has a better correlation of $R2 = 0.81$ with an RMSE of 0.10, without Vsh correction.

The Relationship Between The Errors Are Between Original EZ1 and Full ECZ2



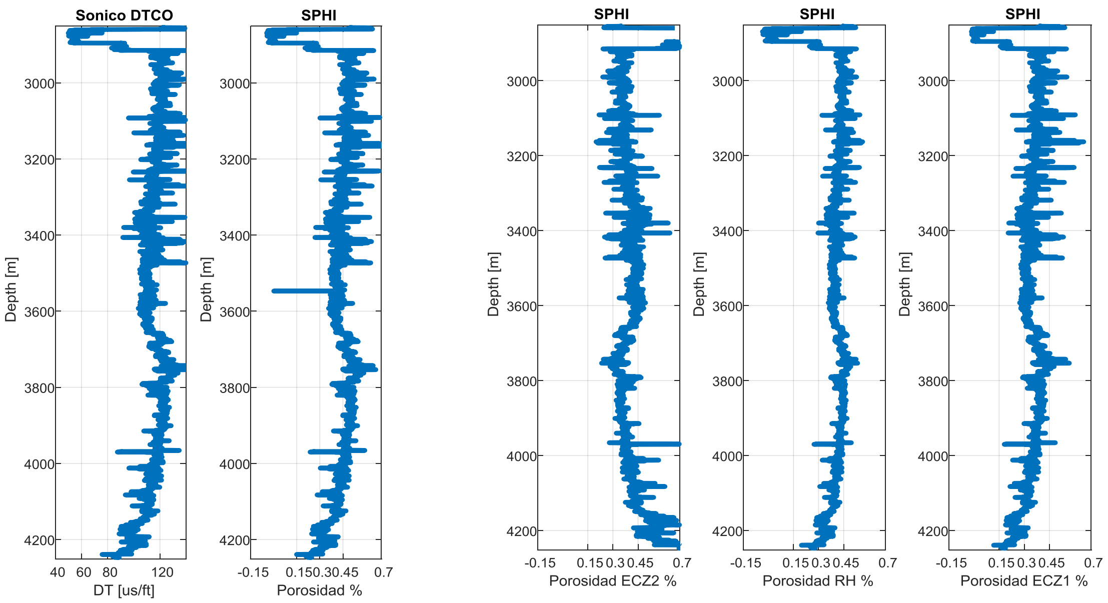
RMSE = 0.0741 R2 =0.8371

Well 2

Matrix Exponent	Fluid Transit Time	Matrix Transit Time
1.63	189	53
DTsh = 140 us/ft	GR min = 20 API and GR max = 50	Proposal Equation Z and Full Proposal Equation Z2

Records, the left image is with the calculated Porosity of the Record that the file has

Table 6



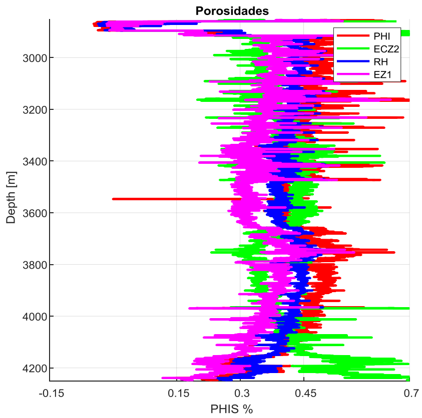
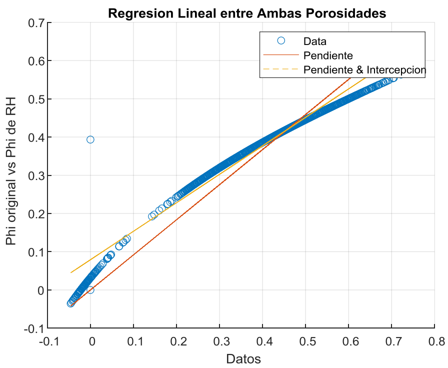


Figure.5Well 2 and its porosities.

Regressions and Errors

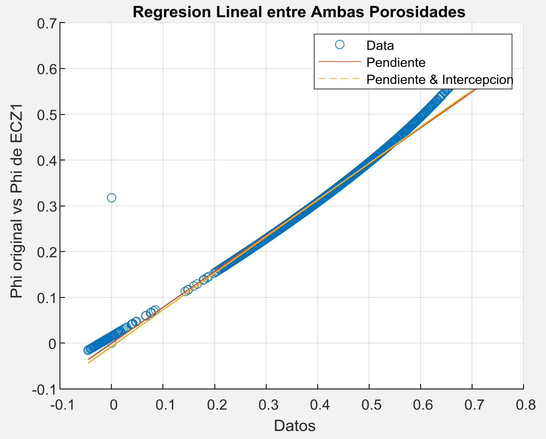
Table 7

Linear Regression between the Original Porosity it comes with and the RH

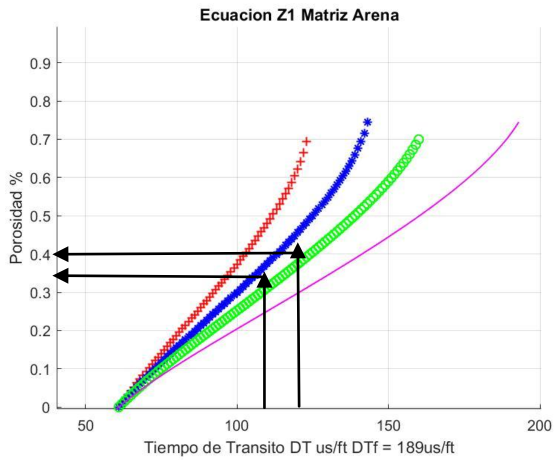


RMSE = 0.0427

Linear Regression between the Original Porosity it comes with and the ECZ1



RMSE = 0.0966



Example at 3800m DT is 120 us/ft with DTsh = 140 us/ft so approx 40%

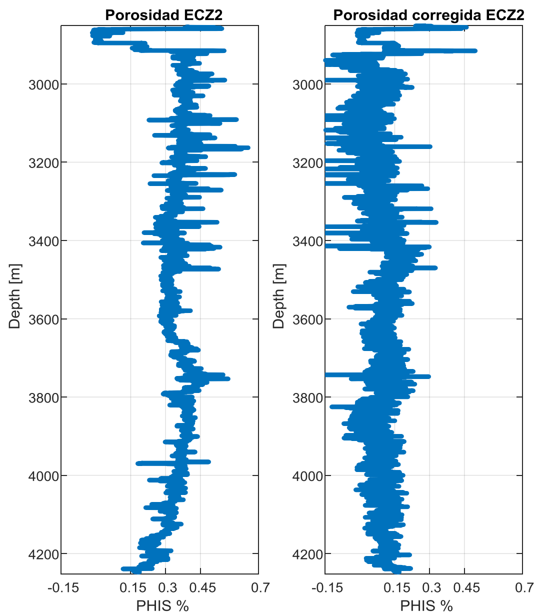
The line in red is from DTsh = 110
The blue line is from DTsh=130
The green line is from DTsh=150
The line in pink is from DTsh=180

Example at 3600m DT is 110 us/ft with DTsh = 140 us/ft so approx 35%

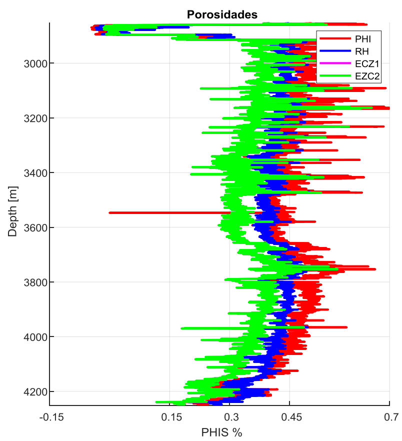
Now in the ECZ2 Expression the complete Equation

$$\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + 1 - \left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]^x = 0$$

Tables are obtained 8, 9 y 10

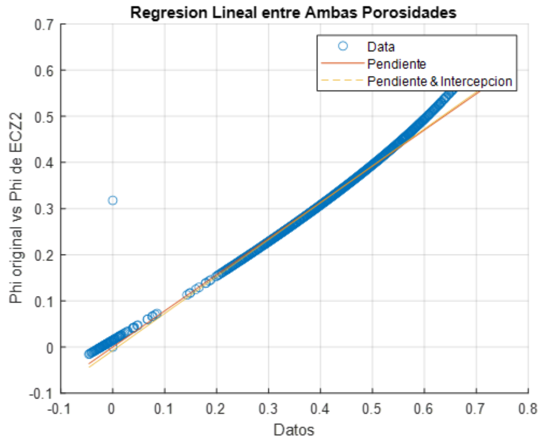


Porosities without Clay Volume correction



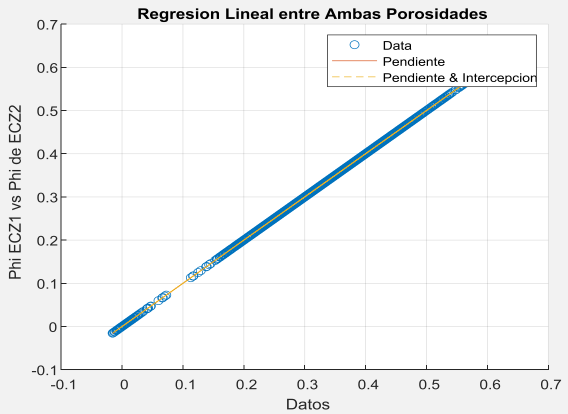
Regressions and Errors

Linear Regression between the Original Porosity with which the log comes and the Complete ECZ2



With high correlation R2= 0.99 and an RMSE of 0.0966

Correlation between the 2 porosities of ECZ1 and ECZ2C



R2 = 1.0
RMSE = 5.8403e-05

Well 3

Matrix Exponent	Fluid Transit Time	Matrix Transit Time
1.63	189	53
DTsh = 110 us/ft	GR min = 60 API and GR max = 120	Proposal Equation Z, RH and Full Proposal Equation Z2

Brief and Previous Interpretation with the graphs of Lithologies.

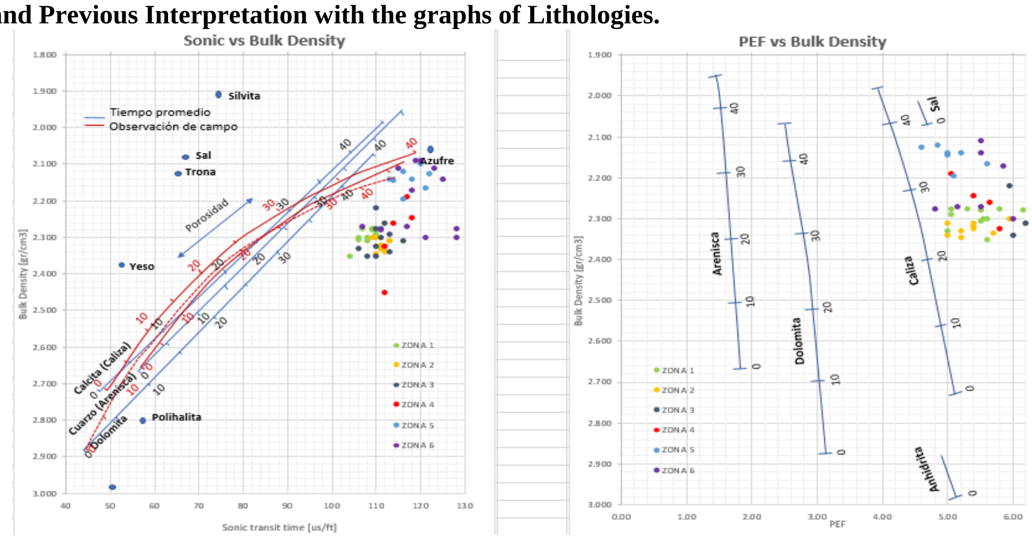
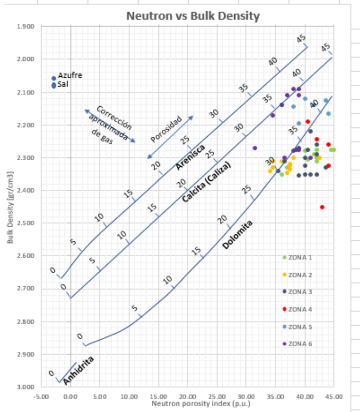
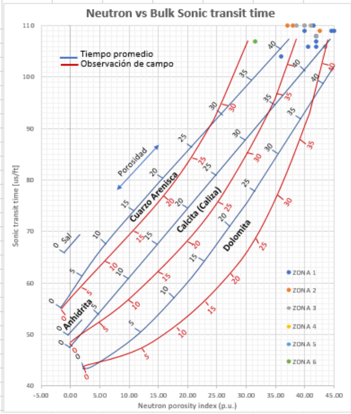
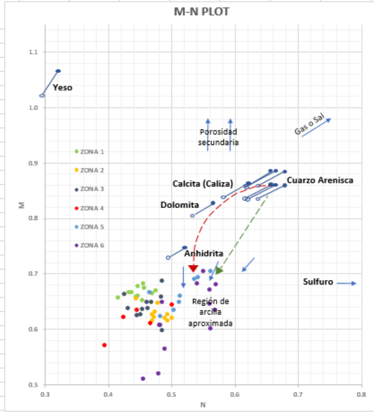


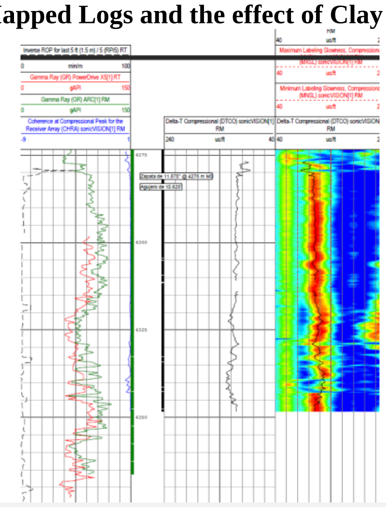
Figure.6 Well 3 and its Lithology.

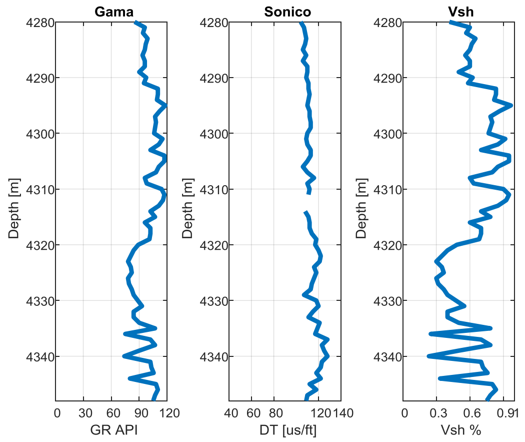


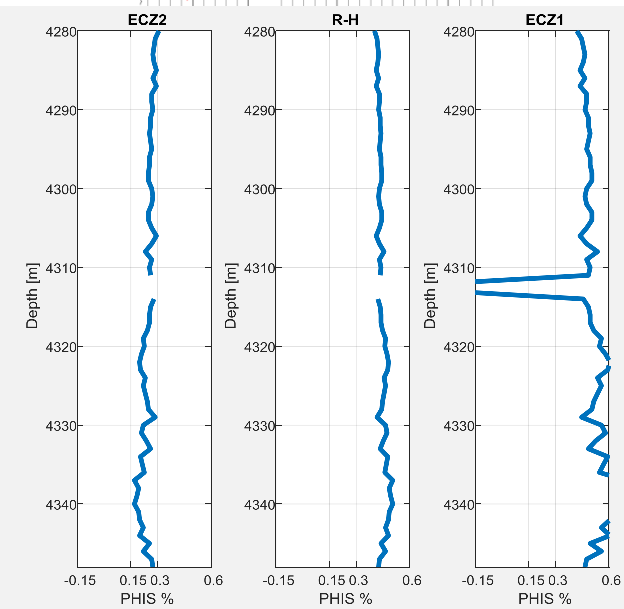


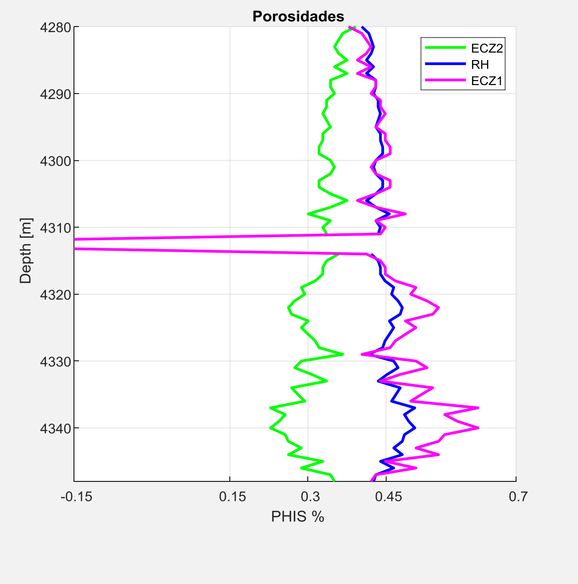


Well 3 Mapped Logs and the effect of Clay

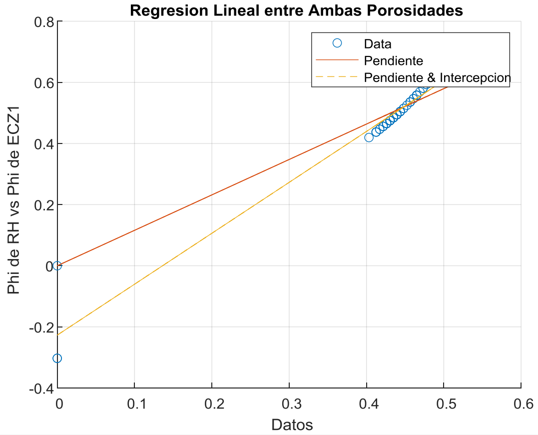






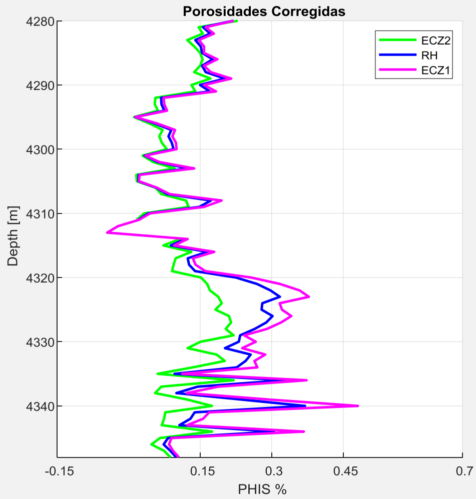


Regressions and Errors



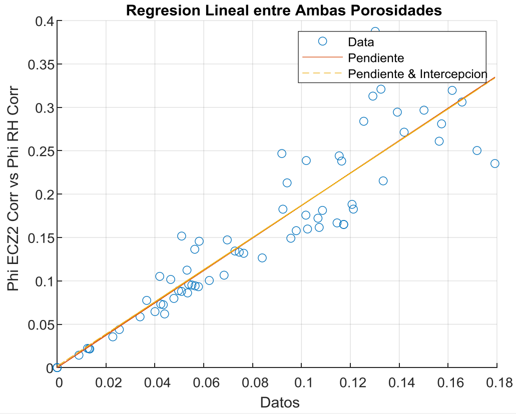
Between the porosities of ECZ2 and RH
RMSE = 0.2034
Between the porosities of RH and ECZ1 there is no relationship and it is observed
RMSE = 0.0925

Now with Vsh Fixes



Regressions and Errors

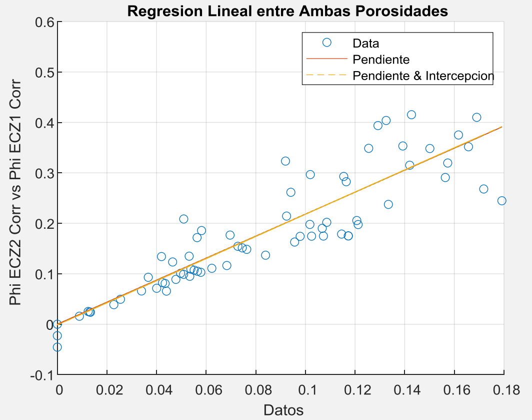
Relationship between ECZ2 and RH corrected



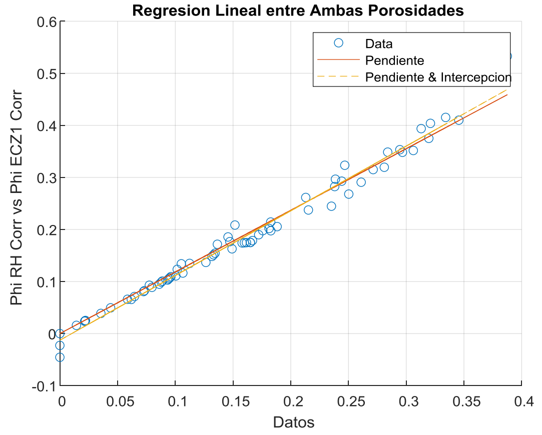
RMSE = 0.0918

Relationship between ECZ2 and ECZ1 corrected

RMSE = 0.1285

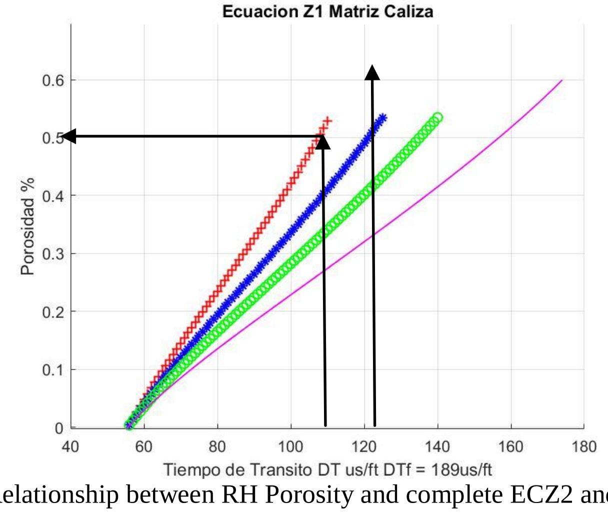


Relationship between HR and ECZ1 corrected



RMSE = 0.0378

A Graphic Form of the ECZ1



Example at 4300m DT is 110 us/ft with DTsh = 110 us/ft so approx 48%

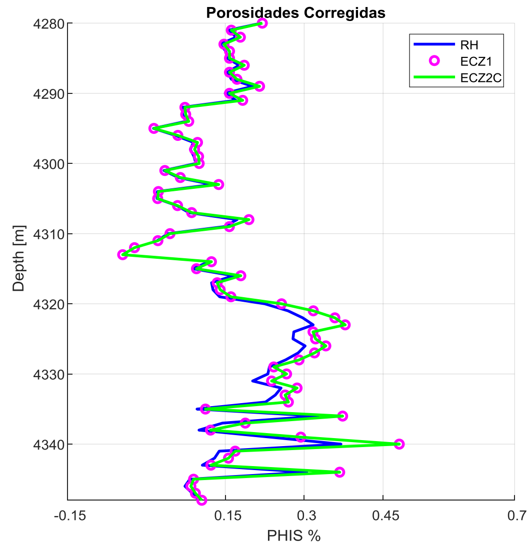
The line in red is from DTsh = 110
The blue line is from DTsh=130
The green line is from DTsh=150
The line in pink is from DTsh=180

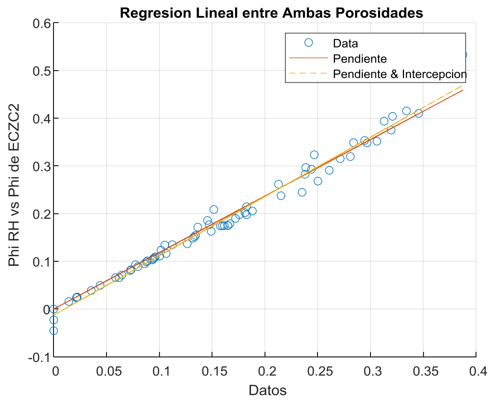
Example at 4340m DT is 125 us/ft with DTsh = 110 us/ft so approx 67%

Relationship between RH Porosity and complete ECZ2 and its correction by Vsh

$$\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + 1 - \left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]^x = 0$$

Is obtained





Relationship between the RH Porosity and the complete Corrected ECZ2 and its Vsh
R2 =0.9841 RMSE = 0.0378

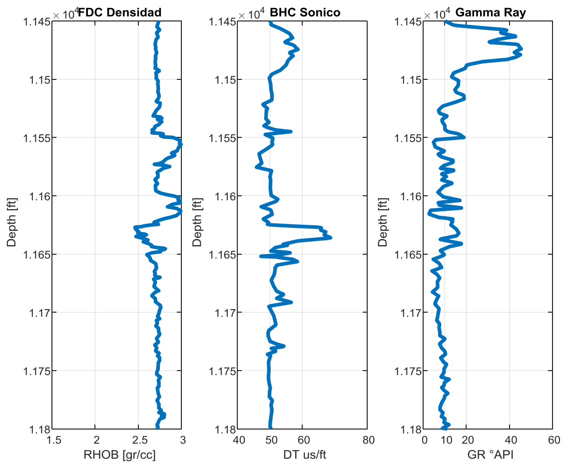
Extra Example with Well 4 and a very low level of Clay

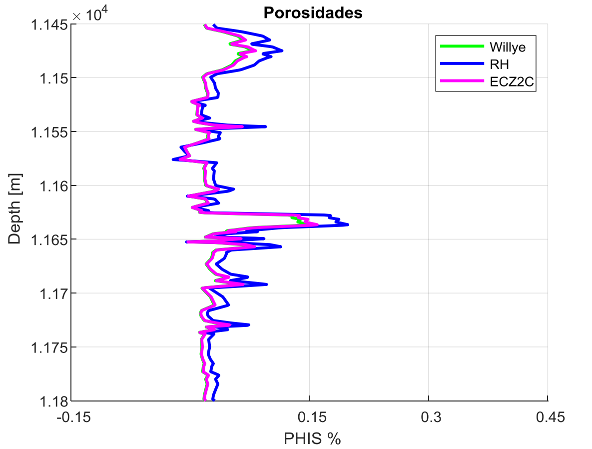
Matrix Exponent	Fluid Transit Time	Matrix Transit Time
1.81	189	47.6
No Clay Time used	Is takenCp = 1	Proposed Equation Z2, RH and Wyllie

Now with Expression 2 directly

$$\emptyset^2 + \left(\frac{\Delta tma}{\Delta tf} - 2\right)\emptyset + 1 - \left[1 - \left(\frac{\Delta tl - \Delta tma}{Cp * (\Delta tf - \Delta tma)}\right)\right]^x = 0$$

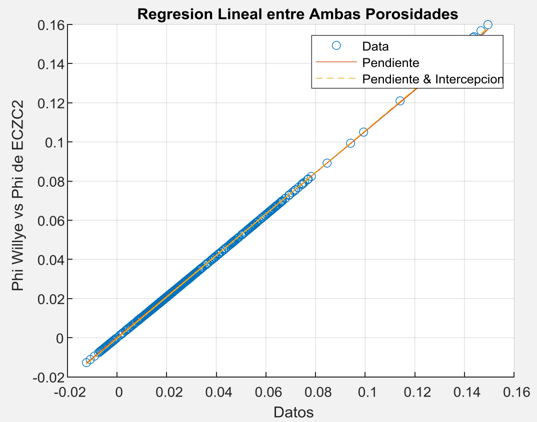
Table 11





The Relationship Between Wyllie Porosities and Full ECZ2

RMSE = 0.0021



Conclusions

The results with the Equations the Proposal and the complete Z2 give good and very accurate approximations with the calculated porosities, we can see that the independent term of the quadratic expression from the Raymer Hunt Equation is very important in the behavior of porosity as an approximation model , in the Complete Equation Z2 within this term we can observe in a certain way the Equation of the Tixier Model and in a certain way Raiga, also the correlations between the approximations have good results and in addition to the RMSE it gave a better low value of the error before the comparisons.

Another important point is the Δtsh which must be chosen according to the experience or criteria of the interpreter as I repeat it again in order to give a better approximation regarding the porosity obtained, for a layer of clean sand (Vsh 10%) Δtsh is replaced with the transit time of the sand for this layer, [see Ref(1)]. The Volume of clay is of importance given that these proposed equations depend on that percentage that is as well seated as possible.

The Lithological Interpretations are found in the Thesis "Porosity Analysis with Sonic Logs and a Comparison with Nuclear Geophysical Logs" of IPN, ESIA -Ticoman Unit 2022 México, by Valeria García Miguel and Osmar Audiel Pacheco López.

Note on the **published Trion Well with an approximate Porosity of 12 to 35% and a Sand Matrix, It is Well 3**

Play	Estilo estructural de la trampa	Litología y ambiente de depósito de la roca almacenadora	Porosidad (%)	Pozos
GP AP E Eoceno	Anticlinales asimétricos con fallas inversas que despegan en sal autóctona	Areniscas y limolitas de canales amalgamados, bancos y desborde de canal	12-35 % (Intergranular y microporosidad secundaria)	Doctus-1, Nobilis-1, Trión-1, Exploratus-1, Maximino-1

Tabla 2.11: Principales características del Play GP AP E Eoceno para el Área Perdido.
Fuente: Comisión Nacional de Hidrocarburos (2019).

Figure.7 Well 3 Data

Source:
http://www.ptolomeo.unam.mx:8080/xmlui/bitstream/handle/132.248.52.100/17446/Tesis.pdf?sequence=7&isAllowed=y
https://rondasmexico.gob.mx/media/1048/atlas_cpp.pdf

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