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Some Properties of the ZJ Transform and application to Partial Differential Equations

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Abstract: The following article is a study, application and results on the ZJ Transform to solve Differential Equations.
Keywords: Laplace Transform, Integral Transform, PDE, ZJ Transform

Introductión

Within the methods to solve Partial Differential Equations there are many methods, as well as those of Transforms and among them is the Laplace Transform, one of the best known and powerful tool, this variation of Transformation is born from the Relation with the Transform of Laplace, by Elzaki and Sumudu [Elzaki Tarig The New Integral Transform "Elzaki Transform" (2011)] in [Fethi Bin Muhammed Belgacem and Ahmed Abdullatif Karaballi. Sumudu transform fundamental properties investigations and applications, 2006] it is possible to obtain said variant of Transformation and in this it is proposed to obtain the Inverse Transform ZJ with the following definition of [Zenteno Jiménez José Roberto November 2022]

Transformed

$$ZJ[f(t),\beta Z)] = ZJ(\beta Z) = \frac{\beta^n}{Z}\int_0^\infty e^{\frac{-Zt}{\beta}} f(t)dt \tag{1}$$

Inverse Transform

$$ZJ^{-1}[f(Z\beta),t)] = \frac{\beta^n Z}{2\pi i}\int_{-\infty}^{\infty} e^{Z\beta t} f\left(\frac{1}{\beta}\right) d\beta Z \tag{2}$$

Relating to other Transforms

Sumudu, Elzaki, Natural, Aboodh, alpha Laplace Integral Transform, Pourreza, Mohand, Sawi, Kamal, G and Complex SEE Transform [See References]

The general Transform would be like, where p and s dependon s in their variation

$$T[f(t),s)] = T(s) = A(s)\int_0^\infty e^{-B(s)t} f(t)dt \tag{3}$$

With

$$|f(t)| < \begin{Bmatrix} Me^{-B(s)t} & t \le 0 \\ Me^{B(s)t} & t \ge 0 \end{Bmatrix}$$

Properties
Linearity

$$ZJ[C1F1(t) + C2F2(t)] = Cf1\left(\frac{Z}{\beta}\right) + C2f2\left(\frac{Z}{\beta}\right)$$

Translation Theorem

$$ZJ[e^{at}F(t)] = f\left(\frac{Z}{\beta} - a\right)$$

Second Translation Theorem

$$ZJ[G(t)] = e^{-\frac{Z}{\beta}a} f\left(\frac{Z}{\beta}\right)$$

With

$$G(t) = \begin{Bmatrix} F(t-a) & t > a \\ 0 & t < a \end{Bmatrix}$$

Example

$$G(t)=\begin{Bmatrix}cos\left(t-\frac{2\pi}{3}\right) & t>a\\ 0 & t<a\end{Bmatrix}$$

$ZJ[cos(t)] = \frac{\beta^{n+1}}{z^2+\beta^2}$ **and** $e^{\frac{-z2\pi}{3\beta}}$ $Thus\ we\ have\ the\ following = \frac{e^{\frac{-z2\pi}{3\beta}}\beta^{n+1}}{z^2+\beta^2}$ **now the inverse holds** $\frac{e^{\frac{-z2\pi}{3\beta}}\beta z}{\beta^2 z^2+1}$

Scale Change

$$ZJ[F(at)] = \frac{1}{a}f\left(\frac{z}{\beta a}\right)$$

Integral

$$ZJ\left[\int_0^\infty F(\tau)d\tau\right] = \frac{f\left(\frac{z}{\beta}\right)}{\frac{z}{\beta}}$$

Convolution Theorem

We define the Convolution of two definite functions of [0, ∞) in the same way as we have done for the Laplace transform. The Convolution f∗g is defined as:

$$f2*f1=\int_0^\infty f2\, f1(t-\tau)d\tau = \frac{\beta^n}{z}F_2\left(\frac{z}{\beta}\right)F_1\left(\frac{z}{\beta}\right)$$

Example 1

$ZJ[e^{-2t}] = ZJ[1]ZJ[e^{-2t}] = \frac{\beta^{n+1}}{z^2} * \frac{\beta^{n+1}}{z(z+2\beta)} = \frac{\beta^{2n+2}}{z^3(z+2\beta)}$ **by Convolution** $\int_0^\infty e^{-2\tau}u(t-\tau)d\tau = \frac{1}{2}(1-e^{-2t})$
now $\frac{1}{2}ZJ[1] - \frac{1}{2}ZJ[e^{-2t}] = \frac{\beta^{2n+2}}{z^3(z+2\beta)}$

Example 2

$ZJ[e^{-3t}]ZJ[e^{t}] = \frac{\beta^{n+1}}{z(z+3\beta)} * \frac{\beta^{n+1}}{z(z-\beta)} = \frac{\beta^{2n+2}}{z^2(z+3\beta)(z-\beta)}$ **by Convolution** $\int_0^\infty e^{-3\tau}e^{t-\tau}d\tau = \frac{1}{4}(e^{t}-e^{-3t})$=
now $\frac{1}{4}ZJ[e^{t}] - \frac{1}{4}ZJ[e^{-3t}] = \frac{\beta^{2n+2}}{z^2(z+3\beta)(z-\beta)}$

PDE Application Examples and Discussion

Now we will see with the following PDE examples

Example 1

$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2}$ **with y(x,0) =0   y (0, t) =1** Using the previous transformation, we have

$$ZJ\left[\frac{dy}{dt}\right] = -y(x,0)\frac{\beta^n}{Z} + \frac{Z}{\beta}\widehat{\varphi}$$

$$ZJ\left[D\frac{\partial^2 y}{\partial x^2}\right] = D\frac{d^2\widehat{\varphi}}{dx^2}$$

Thus we have a Partial Differential Equation of 2 Order with solution

$$\frac{Z}{\beta}\widehat{\varphi} = D\frac{d^2\widehat{\varphi}}{dx^2}$$

$$\widehat{\varphi}(x) = c1e^{\sqrt{\frac{z}{D\beta}}x} + c2e^{-\sqrt{\frac{z}{D\beta}}x}$$

Now the CF **and (0, t) =1** we have $\widehat{\varphi}\left(0,\frac{\beta^n}{z}\right) = \frac{\beta^{n+1}}{z^2}$ using only the negative exponential part as xtends to infinity, we have $\widehat{\varphi}\left(x,\frac{\beta^n}{z}\right) = \frac{\beta^{n+1}}{z^2}e^{-\sqrt{\frac{z}{D\beta}}x}$ Taking the inverse of the previous Expression $f(\frac{1}{\beta})$ and by $z\beta^n$ we directly have the solution

$$\varphi(x,t) = fcer\left(\frac{x}{2\sqrt{t}}\right)$$

Example 2

$\frac{\partial^2 y}{\partial t^2} = D\frac{\partial^2 y}{\partial x^2}$ **with y(x,0) =0** $\frac{\partial y(x,0)}{\partial t} = 0y(0,t) = f(t)$ **and** $\lim_{x\to\infty} y(x,t) = 0$

Using the Transformation $ZJ\left[\frac{\partial^2 y}{\partial t^2}\right] = -\frac{\beta^n}{z} y'(x,0) - \frac{\beta^n}{\beta} y(x,0) + \frac{z^2}{\beta^2}\hat{\varphi}$ **and** $ZJ\left[\frac{\partial^2 y}{\partial x^2}\right] = \frac{d^2\hat{\varphi}}{dx^2}$

$$\left(\frac{Z}{\beta}\right)^2 \hat{\varphi} = D\frac{d^2\hat{\varphi}}{dx^2}$$

Applying the Boundary Conditions and reducing we have $\hat{\varphi}\left(x,\frac{\beta^n}{z}\right) = f\left(\frac{\beta^n}{z}\right) e^{-\frac{z}{D\beta}x}$

Taking the inverse of the previous Expression $f(\frac{1}{\beta})$ and by $z\beta^n$ we directly have the solution

$$y(x,t) = f\left(t - \frac{x}{D}\right)$$

Example 3

$\frac{\partial y}{\partial x} + x\frac{\partial y}{\partial t} = 0$ **with y(x,0) =0 y (0, t) = t**

$$ZJ\left[\frac{\partial y}{\partial x}\right] + xZJ\left[\frac{\partial y}{\partial t}\right] = \frac{d\hat{\varphi}}{dx} + x\left(-y(x,0)\frac{\beta^n}{z} + \frac{z}{\beta}\hat{\varphi}\right) = \frac{d\hat{\varphi}}{dx} + \frac{xz}{\beta}\hat{\varphi} = 0$$

The condition is

$$ZJ[t] = \frac{\beta^{n+2}}{z^3}$$

So

$$\hat{\varphi}\left(x,\frac{\beta^n}{z}\right) = \frac{\beta^{n+2}}{z^3} e^{-\frac{zx^2}{\beta 2}}$$

Taking the inverse of the previous Expression $f(\frac{1}{\beta})$ and by $z\beta^n$ we directly have the solution

$$y(x,t) = \left(t - \frac{x^2}{2}\right)\left(u\left(t - \frac{x^2}{2}\right)\right)$$

Example 4

$\frac{\partial^2 y}{\partial t^2} + a\frac{\partial y}{\partial t} + by = D\frac{\partial^2 y}{\partial x^2}$ **with y(x,0)=α and** $\frac{\partial y(x,0)}{\partial t} = \beta^*$ Using the transformation we have

$$\frac{d^2\hat{\varphi}}{dx^2} - \frac{1}{D}\left(\frac{z^2}{\beta^2} + a\frac{z}{\beta} + b\right)\hat{\varphi} = -\frac{1}{D}\left(\beta^*\frac{\beta^n}{z} + \alpha\frac{\beta^n}{z} + a\alpha\frac{\beta^n}{z}\right)$$

So the particular solution is

$$\hat{\varphi}\left(x,\frac{\beta^n}{z}\right) = \frac{\beta^*\frac{\beta^n}{z} + \alpha\frac{\beta^n}{\beta} + a\alpha\frac{\beta^n}{z}}{-D\left(\frac{d^2}{dx^2}\right) + \frac{z^2}{\beta^2} + a\frac{z}{\beta} + b}$$

Example 5

$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2} + 2\frac{\partial y}{\partial t} + y$**with y(x,0)=$e^x$ and**$\frac{\partial y(x,0)}{\partial t} = -2e^x$ Using the transformation we have

$$\hat{\varphi}\left(x,\frac{\beta^n}{z}\right) = \frac{e^x\frac{\beta^n}{\beta}}{\frac{z^2}{\beta^2} + 2\frac{z}{\beta}}$$

So you can see this $F\left(\frac{\beta^n}{z}\right) G(x) = \frac{\frac{\beta^n}{\beta}}{\frac{z^2}{\beta^2}+2\frac{z}{\beta}} e^x$and tsking$\varphi = G(x)ZJ^{-1}\left[F\left(\frac{\beta^n}{z}\right)\right]$ we have $f(\frac{1}{\beta})$ and by $z\beta^n$ we directly have the solution

$$y(x,t) = e^{-2t} * e^x$$

Example 6

$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2} + 4\frac{\partial y}{\partial t} + 4y$**with y(x,0)=$e^x$ and**$\frac{\partial y(x,0)}{\partial t} = -e^x$ Using the transformation we have

So you can see this $F\left(\frac{\beta^n}{z}\right) G(x) = \frac{\beta}{z}\left(\frac{z\beta^n+3\beta^{n+1}}{z^2+4z\beta+3\beta^2}\right) e^x$ and taking $\varphi = G(x)ZJ^{-1}\left[F\left(\frac{\beta^n}{z}\right)\right]$ we have $f(\frac{1}{\beta})$ and by $z\beta^n$ we directly have the solution

$$y(x,t) = e^{-2t}(cosh(t) + senh(t)) * e^x$$

Example 7

$a^2 \frac{\partial^2 y}{\partial x^2} + Ax = \frac{\partial^2 y}{\partial t^2}$ **with y(x,0)=0 y(0,t)=0 y(1,t)=0 and** $\frac{\partial y(x,0)}{\partial t} = 0$ using the transformation, the c it is because we do not have the second derivative condition in t

$$\hat{\varphi}\left(x, \frac{\beta^n}{z}\right) = \frac{\beta^3}{z^2}\left(\frac{-\beta^n cz - Ax\beta^{n+1}}{a^2\beta^3 - z^3}\right)$$

$f(\frac{1}{\beta})$ and by$z\beta^n$ we directly have the solution

$$y(x,t) = \frac{\left(e^{a^{\frac{2}{3}}t} + 2e^{\frac{-a^{\frac{2}{3}}t}{2}} cos\left(\frac{1}{2}\sqrt{3}a^{\frac{2}{3}}t\right) - 3\right)Ax}{3a^2}$$

with c and c = b

$$-\frac{x}{a^2} - \Big(4\, e^{-1/2\, a^{2/3}\, t}\Big(-a^{2/3}\, b\, e^{3/2\, a^{2/3}\, t} + \sqrt{3}\; a^{2/3}\, b \sin\Big(\frac{1}{2}\sqrt{3}\; a^{2/3}\, t\Big) + a^{2/3}\, b \cos\Big(\frac{1}{2}\sqrt{3}\; a^{2/3}\, t\Big) - x\, e^{3/2\, a^{2/3}\, t} - 2\, x \cos\Big(\frac{1}{2}\sqrt{3}\; a^{2/3}\, t\Big)\Big)\Big) \Big/ \big((\sqrt{3} + -3\, i)(\sqrt{3} + 3\, i)\, a^2\big)$$

Example 8

$\frac{\partial y}{\partial t} = k\frac{\partial^2 y}{\partial x^2} - \frac{a}{L}\frac{\partial y}{\partial x}$**with y(x,0)=** $\frac{L}{ak}\left(1 - e^{-a\left(1-\frac{x}{L}\right)}\right)$ **and y(0,t)=0 y(L,t)=0. With the particular solution as and**$f(x,0) = y(x,0)$

$$\hat{\varphi}\left(x, \frac{\beta^n}{z}\right) = \frac{f(x,t)\frac{\beta^n}{z}}{-k\left(\frac{d^2}{dx^2}\right) + \frac{a}{L}\frac{d}{dx} + \frac{z}{\beta}}$$

The Transformed is

$$\hat{\varphi}\left(x, \frac{\beta^n}{z}\right) = \frac{f(x,t)L\beta^{n+1}}{z(zL + (a - kL)\beta)}$$

$f(\frac{1}{\beta})$ and by$z\beta^n$ we directly have the solution $\hat{\varphi} = f(x,t)ZJ^{-1}\left[F\left(\frac{\beta^n}{z}\right)\right]$

$y(x,t) = \frac{L}{ak}\left(1 - e^{-a\left(1-\frac{x}{L}\right)}\right)e^{-\left(\frac{a-kL}{L}\right)t}$ Satisfying the Initial and Boundary Conditions

Example 9

Application of an Oil Well that produces constant flow in an infinite reservoir with the online source solution

$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = \frac{1}{\eta}\frac{\partial P}{\partial t}$ **with**$0 < r < \infty$ $p(r,0) = Pi$ $\lim_{r\to 0}\left(r\frac{\partial P}{\partial r}\right) = \frac{-q\mu}{2\pi kh}\lim_{r\to\infty} p(r,t) = Pi$

Taking the following transformation

$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P}{\partial r_D}\right) = \frac{\partial P}{\partial t_D}$ **with** $0 < r_D < \infty$ $P_D(r_D,0) = 0$ $\lim_{r_D\to 0}\left(r_D\frac{\partial P}{\partial r_D}\right) = -1\lim_{r_D\to\infty} P_D(r_D,t_D) = 0$

applying the transformation to the temporary part we have and to the boundary conditions

$$\frac{1}{r_D}\frac{d}{dr_D}\left(r_D\frac{dP}{dr_D}\right) = \frac{-P_D\beta^n}{z} + \frac{zP_D}{\beta}$$

Which is a Bessel equation with the solution of the modified Bessel functions of 1 and 2 class

$$P\left(r_D, \frac{\beta^n}{z}\right) = AI_0\left(\sqrt{\frac{\beta^n}{z}}r_D\right) + BK_0\left(\sqrt{\frac{\beta^n}{z}}r_D\right)$$

Now with the CF and the properties of the Bessel functions we obtain

$$P\left(r_D, \frac{\beta^n}{z}\right) = \frac{\beta^{n+1}}{z^2}K_0\left(\sqrt{\frac{z}{\beta}}r_D\right)$$

$f\left(\frac{1}{\beta}\right)$ and by the Convolution Theorem and by $z\beta^n$ we directly have the solution

$$P\left(r_D, \frac{\beta^n}{z}\right) = \frac{1}{z\beta}K_0(\sqrt{z\beta}r_D)$$

The Convolution with the Transform, now there is a unit step $f1 = \frac{1}{z\beta} = f1(t_D) = u(t_D)$ y $f2 = K_0(\sqrt{z\beta}r_D) = \frac{e^{-\frac{r_D^2}{4t_D}}}{2t_D}$y

$$P(r_D, t_D) = \frac{1}{2}\int_0^{t_D} u\,(t_D - \tau)\frac{e^{-\frac{r_D^2}{4\tau}}}{\tau}d\tau = \frac{1}{2}E_1\left(\frac{r_D^2}{4t_D}\right)$$

Thus, as seen in the previous examples, there is another variant or Transform linked to Laplace, among others such as Natural, Aboodh, Kashuri – Fundo, Srivastava, ZZ, Ramadan Group [R. I. Nuruddeena, Lawal Muhammadb, A.M. Nassc, T. A. Sulaiman. A Review of the Integral Transforms-Based Decomposition Methods and their Applications in Solving Nonlinear PDEs] and the Complex SEE [Eman A. Mansour et al 2021 Application of New Transform "Complex SEE Transform" to Partial Differential Equations]. Also the properties that it has which are linked to Laplace and a new table with some of the basic functions and their transformation.

ZJ Transformed

$$ZJ[f(t), \beta Z)] = ZJ(\beta Z) = \frac{\beta^n}{Z}\int_0^{\infty} e^{\frac{-Zt}{\beta}} f(t)dt$$

ZJ Inverse Transformed

$$ZJ^{-1}[f(Z\beta), t)] = \frac{\beta^n Z}{2\pi i}\int_{-\infty}^{\infty} e^{Z\beta t} f\left(\frac{1}{\beta}\right) d\beta Z$$

Table 1 of some transformations

1	$\frac{\beta^{n+1}}{z^2}$
t	$\frac{\beta^{n+2}}{z^3}$
t^2	$\frac{2\beta^{n+3}}{z^4}$
t^m	$\frac{m!\ \beta^{n+(m+1)}}{z^{m+2}}$
$\sqrt{t}$	$\frac{\sqrt{\pi}\beta^{n+3/2}}{2z^{5/2}}$
$\frac{1}{\sqrt{t}}$	$\frac{\sqrt{\pi}\beta^{n+1/2}}{z^{3/2}}$
e^{at}	$\frac{\beta^{n+1}}{z(z-a\beta)}$
$sen(bt)$	$\frac{b\beta^{n+2}}{z(z^2+z^2\beta^2)}$
$cos(bt)$	$\frac{\beta^{n+1}}{z^2+b^2\beta^2}$
$t^n e^{kt}$	$\frac{n!\,\beta^n}{\left(k-\frac{z}{\beta}\right)^{n+1} z}$
$senh(bt)$	$\frac{b\beta^{n+2}}{z(z^2-z^2\beta^2)}$
$cosh(bt)$	$\frac{\beta^{n+1}}{(z^2-z^2\beta^2)}$
$\delta(t-t_0)$	$\frac{\beta^n e^{-\frac{z}{\beta}t0}}{z}$
$J_0(t)$	$\frac{\beta^{n+1}}{z\sqrt{\beta^2+z^2}}$

$J_1(t)$	$\frac{\beta^n\left(\sqrt{\beta^2+z^2}-z\right)}{z\sqrt{\beta^2+z^2}}$
$J_n(at)$	$\frac{\beta^n\left(\sqrt{a^2\beta^2+z^2}-z\right)^n}{a^2z\sqrt{a^2\beta^2+z^2}}$
$\frac{dy}{dt}$	$-y(x,0)\frac{\beta^n}{z}+\frac{z}{\beta}\widehat{\varphi}$
$\frac{d^2y}{dt^2}$	$-\frac{\beta^n}{z}y'(x,0)-\frac{\beta^n}{\beta}y(x,0)+\frac{z^2}{\beta^2}\widehat{\varphi}$
$\frac{d^3y}{dt^3}$	$-\frac{\beta^n}{z}y''(x,0)-\frac{\beta^n}{\beta}y'(x,0)-\frac{\beta^n z}{\beta^2}y(x,0)+\frac{z^3}{\beta^3}\widehat{\varphi}$
$f(t)^n$	$\frac{z^n\widehat{\varphi}}{\beta^n}-\frac{\beta^n}{z}\left(\sum_{k=1}^{n-1}\left(\frac{z}{\beta}\right)^{n-1-k}y^k(0)\right)$
$\frac{\partial^2 y}{\partial x^2}$	$\frac{d^2\widehat{\varphi}}{dx^2}$
$\frac{\partial y}{\partial x}$	$\frac{d\widehat{\varphi}}{dx}$

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