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Application of the ZJ Transform to Integral Equations type Volterra – Fredholm and Integro Differentials

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Abstract: The following article is a continuation and is a study, application and results on the ZJ Transform to solve Volterra, Fredholm and Integro differential type Integral Equations

Keywords: ZJ Transform, Volterra Integral Equation, Fredholm Integral Equation, Integral Differential Equations

Introduction

The following study is a continuation of the application of the ZJ Transform [16,17] but now to Integral Equations, which we are going to address with examples linked to solving these following cases, first we will remember the Convolution Theorem and hence the properties of the first derivative differential equations.

ZJ Transform	ZJ Inverse Transform
$ZJ[\beta Z] = \frac{\beta^n}{z}\int_0^\infty e^{-\frac{z}{\beta}t} f(t)dt$	$ZJ[\beta Z]^{-1} = \frac{z\beta^n}{2\pi i}\int_{-\infty}^{\infty} e^{Z\beta t} f\left(\frac{1}{\beta}\right) d\beta z$

Now from the Convolution Theorem we have the following

Convolution Theorem we define the Convolution of two definite functions of [0, ∞) in the same way as we have done for the Laplace transform.

Convolution Theorem

Let f1 and f2 have a new Integral transformation F1 and F2, then the new integral transform of the Convolution of f1 and f2 is:

$$ZJ[f1 * f2] = \frac{\beta^n}{z}\int_0^\infty e^{-\frac{z}{\beta}t}\int_0^\infty f1(t)f2(t-\tau)\,dt$$

$$ZJ[f1 * f2] = \frac{\beta^n}{z}\int_0^\infty f1(\tau)d\tau\int_0^\infty e^{-\frac{z}{\beta}t} f2(t-\tau)\,dt$$

$$ZJ[f1 * f2] = \frac{\beta^n}{z}\int_0^\infty e^{-\frac{z}{\beta}t} f1(\tau)d\tau\int_0^\infty e^{-\frac{z}{\beta}t} f2(t)\,dt$$

$$ZJ[f1 * f2] = \frac{F1\left(\frac{z}{\beta}\right)F2\left(\frac{z}{\beta}\right)}{\frac{\beta^n}{z}}$$

The Convolution f∗g is defined as:

$$f1 * f2 = \int_0^\infty f1(t)f2(t-\tau)d\tau = \frac{F2\left(\frac{z}{\beta}\right)F1\left(\frac{z}{\beta}\right)}{\frac{\beta^n}{z}}$$

By completing the previous examples on Convolution in [17] we have this

$$y(t) = \int_0^t f(u)g(t-u)du$$

$$iff = e^t andg = e^{2t} soy(t) = \int_0^t e^u e^{2(t-u)}du = e^{2t} - e^t$$

$$=\frac{\beta^{n+1}}{z(z-2\beta)}-\frac{\beta^{n+1}}{z(z-\beta)}=\frac{\beta^{n+1}z^2-z\beta^{n+2}-\beta^{n+1}z^2+2z\beta^{n+2}}{z^2(z-2\beta)(z-\beta)}=\frac{z\beta^{n+2}}{z^2(z-2\beta)(z-\beta)}=\frac{\beta^{n+2}}{z(z-2\beta)(z-\beta)}$$

By the transform we have:

$$ZJ[e^t]ZJ[e^{2t}]=\left[\frac{\beta^{n+1}}{z(z-2\beta)}*\frac{\beta^{n+1}}{z(z-\beta)}\right]=\left[\frac{\beta^{2n+2}}{z^2(z-2\beta)(z-\beta)}\right]$$

$$y\left(\frac{z}{\beta^n}\right)\left(\frac{\beta^{2n+2}}{z^2(z-2\beta)(z-\beta)}\right)=\frac{\beta^{n+2}}{z(z-2\beta)(z-\beta)}$$

Other Example is:

$$f=e^{-2t}yg=1\ \ soy(t)=\int_0^t e^{-2u}u\,(t-\tau)\,d\tau=\frac{1}{2}(1-e^{-2t})$$

$$\text{and with}\frac{1}{2}ZJ[1]-\frac{1}{2}ZJ[e^{-2t}]=$$

$$\left(\frac{\beta^{n+1}}{z^2}-\frac{\beta^{n+1}}{z(z+2\beta)}\right)\left(\frac{1}{2}\right)=\left(\frac{z^2\beta^{n+1}+2z\beta^{n+2}-z^2\beta^{n+1}}{z^3(z+2\beta)}\right)\left(\frac{1}{2}\right)=\frac{2z\beta^{n+2}}{z^3(z+2\beta)}\left(\frac{1}{2}\right)=\frac{\beta^{n+2}}{z^2(z+2\beta)}$$

$$\text{and}ZJ[e^{-2t}]ZJ[1]=\frac{\beta^{n+1}}{z(z+2\beta)}*\frac{\beta^{n+1}}{z^2}=\frac{\beta^{2n+2}}{z^3(z+2\beta)}\left(\frac{z}{\beta^n}\right)=\frac{\beta^{n+2}}{z^2(z+2\beta)}$$

Applications to Integral Equations

Let's go with some application examples, applying the ZJ Transform to the following Integral Equations we have:

Example 1

$$u(t)=t+\int_0^t(t-\tau)u(\tau)d\tau\ \text{consol.}\quad y=\text{senh}(t)$$

$$\widehat{\varphi}=\frac{\beta^{n+2}}{z^3}+\left(\frac{z}{\beta^n}\right)\left(\frac{\beta^{n+2}}{z^3}\right)\widehat{\varphi}\rightarrow\widehat{\varphi}=\frac{\beta^{n+2}}{z^3}+\frac{\beta^2}{z^2}\widehat{\varphi}$$

$$\left[1-\frac{\beta^2}{z^2}\right]\widehat{\varphi}=\frac{\beta^{n+2}}{z^3}\rightarrow\left[\frac{z^2-\beta^2}{z^2}\right]\widehat{\varphi}=\frac{\beta^{n+2}}{z^3}\rightarrow\widehat{\varphi}=\frac{\frac{\beta^{n+2}}{z^3}}{\frac{z^2-\beta^2}{z^2}}$$

$$\widehat{\varphi}=\frac{\beta^{n+2}z^2}{(z^2-\beta^2)z^3}=\frac{\beta^{n+2}}{z(z^2-\beta^2)}\ \sim f\left(\frac{1}{\beta}\right)$$

$$\widehat{\varphi}=\frac{1}{z\beta^{n+2}\left(z^2-\frac{1}{\beta^2}\right)}=\frac{1}{z\beta^{n+2}\left(\frac{z^2\beta^2-1}{\beta^2}\right)}$$

$$\widehat{\varphi}=\frac{\beta^2}{z\beta^{n+2}(z^2\beta^2-1)}=\frac{1}{z\beta^n(z^2\beta^2-1)}(z\beta^n)$$

$$\widehat{\varphi}=\frac{z\beta^n}{z\beta^n(z^2\beta^2-1)}=\frac{1}{(z^2\beta^2-1)}$$

$$\rightarrow\text{now the inverses enh }(t)$$

Example 2

$$y(t)=1-\int_0^x(t-\tau)y(\tau)d\tau$$

$$\widehat{\varphi}=\frac{\beta^{n+1}}{z^2}-\left(\frac{\beta^n}{z}\right)\left[\frac{\beta^{n+2}}{z^3}\right]\widehat{\varphi}\rightarrow\text{si}\frac{z}{\beta^n}\sim-\left[\frac{\beta^2}{z^2}\right]\widehat{\varphi}$$

$$\widehat{\varphi}+\left[\frac{\beta^2}{z^2}\right]\widehat{\varphi}=\frac{\beta^{n+1}}{z^2}\rightarrow\left[1+\frac{\beta^2}{z^2}\right]\widehat{\varphi}=\frac{\beta^{n+1}}{z^2}$$

$$\frac{z^2+\beta^2}{z^2}\widehat{\varphi}=\frac{\beta^{n+1}}{z^2}\rightarrow\widehat{\varphi}=\frac{\frac{\beta^{n+1}}{z^2}}{\frac{z^2+\beta^2}{z^2}}=\frac{\beta^{n+1}}{z^2+\beta^2}\text{now the inverse}$$

$$\widehat{\varphi} = \frac{\frac{1}{\beta^{n+1}}}{z^2 + \frac{1}{\beta^2}} = \frac{\frac{1}{\beta^{n+1}}}{\frac{z^2\beta^2+1}{\beta^2}} = \frac{\beta^2}{\beta^{n+1}(z^2\beta^2+1)} = \frac{\beta}{\beta^n(z^2\beta^2+1)}(z\beta^n)$$

$$\widehat{\varphi} = \frac{z\beta}{(z^2\beta^2+1)} \rightarrow \text{ with solution like } \cos(t)$$

Example 3

$$u(t) = 3t - 9\int_0^t (t-\tau)y(\tau)d\tau$$

$$\widehat{\varphi} = 3\frac{\beta^{n+2}}{z^3} - 9\left(\frac{z}{\beta^n}\right)\left(\frac{\beta^{n+2}}{z^3}\right)\widehat{\varphi} \rightarrow \widehat{\varphi} = \frac{3\beta^{n+2}}{z^3} - 9\frac{\beta^2}{z^2}\widehat{\varphi}$$

$$\left[1 + \frac{9\beta^2}{z^2}\right]\widehat{\varphi} = 3\frac{\beta^{n+2}}{z^3} \rightarrow \left[\frac{z^2+9\beta^2}{z^2}\right]\widehat{\varphi} = \frac{3\beta^{n+2}}{z^3} \rightarrow \widehat{\varphi} = \frac{\frac{3\beta^{n+2}}{z^3}}{\frac{z^2+9\beta^2}{z^2}}$$

$$\widehat{\varphi} = \frac{3\beta^{n+2}z^2}{z^3(z^2+9\beta^2)} = \frac{3\beta^{n+2}}{z(z^2+9\beta^2)} \sim f\left(\frac{1}{\beta}\right)$$

$$\widehat{\varphi} = \frac{\frac{3}{\beta^{n+2}}}{z\left(z^2+\frac{9}{\beta^2}\right)} = \frac{\frac{3}{\beta^{n+2}}}{z\left(\frac{z^2\beta^2+9}{\beta^2}\right)} = \frac{3\beta^2}{z\beta^{n+2}(z^2\beta^2+9)} = \frac{3}{z\beta^n(z^2\beta^2+9)}(z\beta^n)$$

$$\widehat{\varphi} = \frac{3}{(z^2\beta^2+9)} \rightarrow$$

with solution like ~ sen(3t)

Example 4

$$u(t) = 1 - 4\int_0^t (t-\tau)y(\tau)d\tau$$

$$\widehat{\varphi} = \frac{\beta^{n+1}}{z^2} - 4\left(\frac{z}{\beta^n}\right)\left(\frac{\beta^{n+2}}{z^3}\right)\widehat{\varphi} \rightarrow \widehat{\varphi} = \frac{\beta^{n+1}}{z^2} - 4\frac{\beta^2}{z^2}\widehat{\varphi}$$

$$\left[1 + \frac{4\beta^2}{z^2}\right]\widehat{\varphi} = \frac{\beta^{n+1}}{z^2} \rightarrow \left[\frac{z^2+4\beta^2}{z^2}\right]\widehat{\varphi} = \frac{\beta^{n+1}}{z^2}$$

$$\widehat{\varphi} = \frac{\frac{\beta^{n+1}}{z^2}}{\frac{z^2+4\beta^2}{z^2}} = \frac{\beta^{n+1}z^2}{z^2(z^2+4\beta^2)} = \frac{\beta^{n+1}}{z^2+4\beta^2} \quad \sim \quad f\left(\frac{1}{\beta}\right)$$

$$\rightarrow \frac{1}{\beta^{n+1}\left(z^2+\frac{4}{\beta^2}\right)} \rightarrow \frac{1}{\beta^{n+1}\left(\frac{z^2\beta^2+4}{\beta^2}\right)}$$

$$\rightarrow \frac{1}{\beta^{n+1}\left(\frac{z^2\beta^2+4}{\beta^2}\right)} \rightarrow \frac{\beta^2}{\beta^{n+1}(z^2\beta^2+4)} \rightarrow \frac{\beta}{\beta^n(z^2\beta^2+4)}(z\beta^n)$$

$$\rightarrow \frac{z\beta}{(z^2\beta^2+4)} \rightarrow \text{ with solution like} \sim \cos(2t)$$

Example 5

$$u(x) = 6x - x^3 + \frac{1}{2}\int_0^x tu(t)dt$$

$$\widehat{\varphi} = (6x - x^3)\left(\frac{\beta^{n+1}}{z^2}\right) + \frac{1}{2}\left(\frac{z}{\beta^n}\right)\left(\frac{\beta^{n+2}}{z^3}\right)\widehat{\varphi} \rightarrow \widehat{\varphi} = b\frac{\beta^{n+1}}{z^2} + \frac{1}{2}\frac{\beta^2}{z^2}\widehat{\varphi} \rightarrow \widehat{\varphi} - \frac{1}{2}\frac{\beta^2}{z^2}\widehat{\varphi} = b\frac{\beta^{n+1}}{z^2}$$

$$\left[1 - a\frac{\beta^2}{z^2}\right]\widehat{\varphi} = b\frac{\beta^{n+1}}{z^2} \rightarrow \widehat{\varphi} = \frac{b\frac{\beta^{n+1}}{z^2}}{\left[1 - a\frac{\beta^2}{z^2}\right]} = \frac{\frac{b\beta^{n+1}}{z^2}}{\frac{z^2-a\beta^2}{z^2}} = \frac{b\beta^{n+1}}{z^2 - a\beta^2} \rightarrow f\left(\frac{1}{\beta}\right)$$

$$\widehat{\varphi} = \frac{\frac{b}{\beta^{n+1}}}{z^2 - \frac{a}{\beta^2}} = \frac{\frac{b}{\beta^{n+1}}}{\frac{z^2\beta^2-a}{\beta^2}} = \frac{b\beta^2}{\beta^{n+1}(z^2\beta^2-a)} = \frac{b\beta}{\beta^n(z^2\beta^2-a)}(z\beta^n)$$

$$\widehat{\varphi} = \frac{bz\beta}{(z^2\beta^2 - a)} \to \frac{bs}{(s^2 - a)} \text{ and this is how } \cosh\left(\frac{x}{\sqrt{2}}\right)$$

Now how

$$(6x - x^3)\left[\cosh\left(\frac{x}{\sqrt{2}}\right)\right] = (6x - x^3)\left[1 + \frac{\left(\frac{x}{\sqrt{2}}\right)^2}{2!} + \frac{\left(\frac{x}{\sqrt{2}}\right)^4}{4!} + \cdots\right]$$

$$= 6x - x^3 + (6x - x^3)\frac{\left(\frac{x}{\sqrt{2}}\right)^2}{2!} + (6x - x^3)\frac{\left(\frac{x}{\sqrt{2}}\right)^4}{4!} + \cdots$$

$$= 6x - x^3 + \frac{6x^3}{2 * 2!} - \frac{x^5}{2 * 2!} + \cdots \text{ so the power terms are small}$$

$$= 6x + \left(\frac{6}{4} - 1\right)x^3 - x^5 + \cdots = 6x \text{is the solution in closed form}$$

For Fredholm Type Equations

Example 1

$$u(x) = \frac{9}{10}x^2 + \int_0^1 \frac{1}{2}x^2t^2u(t)dt$$

$$u(x) = \frac{9}{10}x^2 + \frac{1}{2}x^2\int_0^1 t^2u(t)dt$$

$$\widehat{\varphi} = \frac{9}{10}x^2\frac{\beta^{n+1}}{z^2} + \frac{1}{2}x^2\left(\frac{2\beta^3}{z^3}\right)\widehat{\varphi} \to \widehat{\varphi} = \frac{9}{10}x^2\frac{\beta^{n+1}}{z^2} + x^2\frac{\beta^3}{z^3}\widehat{\varphi}$$

$$\left[1 - x^2\frac{\beta^3}{z^3}\right]\widehat{\varphi} = \frac{9}{10}x^2\frac{\beta^{n+1}}{z^2} \to \left[1 - a\frac{\beta^3}{z^3}\right]\widehat{\varphi} = bx^2\frac{\beta^{n+1}}{z^2}$$

$$\left[\frac{z^3 - a\beta^3}{z^3}\right]\widehat{\varphi} = bx^2\frac{\beta^{n+1}}{z^2} \to \widehat{\varphi} = \frac{bx^2\frac{\beta^{n+1}}{z^2}}{\frac{z^3 - a\beta^3}{z^3}} \to \widehat{\varphi} = \frac{bx^2\beta^{n+1}z}{z^3 - a\beta^3}$$

$$\to \frac{bx^2\beta^{n+1}z}{z^3 - a\beta^3} \to \frac{bx^2z}{\beta^{n+1}\left(z^3 - \frac{a}{\beta^3}\right)} \to \frac{bx^2z}{\beta^{n+1}\left(\frac{z^3\beta^3 - a}{\beta^3}\right)} \to \frac{bx^2z\beta^3}{\beta^{n+1}(z^3\beta^3 - a)}$$

$$\to \frac{bx^2z\beta^2}{\beta^n(z^3\beta^3 - a)} * z\beta^n \to bx^2\frac{z^2\beta^2}{(z^3\beta^3 - a)} \to \text{expanded form}$$

$$= \frac{4e^{\sqrt[3]{\frac{9}{10}}t}}{3 + 9} = \frac{4}{12}e^{\sqrt[3]{\frac{9}{10}}t} \sim \frac{e^{\sqrt[3]{\frac{9}{10}}t}}{3} \sim \frac{1}{3}\left[1 + \sqrt[3]{\frac{9}{10}}t + \frac{\left(\sqrt[3]{\frac{9}{10}}t\right)^2}{2!} + \frac{\left(\sqrt[3]{\frac{9}{10}}t\right)^3}{3!} + \cdots\right]$$

$$\approx \frac{9}{10}x^2\left(\frac{1}{3}\right) \approx \frac{9x^2}{30} \text{ thus } kx^2 = 0.3x^2 \text{or } (k = 1)x^2 = x^2$$

Example 2

$$u(x) = \frac{5}{6}x + \frac{1}{2}\int_0^1 xtu(t)dt \text{ with sol.} \frac{5}{6}x \approx x$$

$$\widehat{\varphi} = \frac{5}{6}x\frac{\beta^{n+1}}{z^2} + \frac{x}{2}\left(\frac{z}{\beta^n}\right)\left(\frac{\beta^{n+2}}{z^3}\right)\widehat{\varphi} \to \widehat{\varphi} = \frac{5}{6}x\left(\frac{\beta^{n+1}}{z^2}\right) + \frac{x}{2}\left(\frac{\beta^2}{z^2}\right)\widehat{\varphi} \to \left[1 - \frac{x}{2}\left(\frac{\beta^2}{z^2}\right)\right]\widehat{\varphi} = \frac{5}{6}x\left(\frac{\beta^{n+1}}{z^2}\right)$$

$$\left[1 - a\left(\frac{\beta^2}{z^2}\right)\right]\widehat{\varphi} = b\left(\frac{\beta^{n+1}}{z^2}\right) \to \left[\frac{z^2 - a\beta^2}{z^2}\right]\widehat{\varphi} = b\left(\frac{\beta^{n+1}}{z^2}\right)$$

$$\widehat{\varphi} = \frac{b\left(\frac{\beta^{n+1}}{z^2}\right)}{\left[\frac{z^2 - a\beta^2}{z^2}\right]} = \frac{b\beta^{n+1}z^2}{z^2(z^2 - a\beta^2)} = \frac{b\beta^{n+1}}{z^2 - a\beta^2} \to f\left(\frac{1}{\beta}\right)$$

$$\widehat{\varphi} = \frac{\frac{b}{\beta^{n+1}}}{z^2 - \frac{a}{\beta^2}} = \frac{\frac{b}{\beta^{n+1}}}{\frac{z^2\beta^2 - a}{\beta^2}} = \frac{b\beta^2}{\beta^{n+1}(z^2\beta^2 - a)} = \left(\frac{b\beta}{\beta^n(z^2\beta^2 - a)}\right)(z\beta^n) = \frac{bz\beta}{(z^2\beta^2 - a)}$$

$$\rightarrow \; = \frac{5}{6} x\left[\frac{s}{s^2-a}\right] = \frac{5}{6} x\cosh(\sqrt{a})t(\sqrt{a}) = cte$$

$$= \frac{5}{6} x\left[1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\right]$$

$$\left[\frac{5}{6}+\frac{1}{8}\right]x = \frac{41}{48} = 0.85$$

$$\therefore 0.85x \approx x$$

$$x = \sqrt{a} = \sqrt{\frac{x}{2}}$$

$$\left(\sqrt{\frac{x}{2}}\right)^2 = \frac{x}{4}$$

$$\left(\sqrt{\frac{x}{2}}\right)^4 = \frac{x^2}{4}$$

For Integro-Differential Equations

Example 1

$$\int_0^t y(\tau)\cos(t-\tau)\,d\tau = y'(t)\,y(0) = 1$$

$$\frac{z}{\beta^n}\hat{\varphi}\left[\frac{\beta^{n+1}}{z^2+\beta^2}\right] = \left[-y(x,0)\frac{\beta^n}{z}+\frac{z}{\beta}\hat{\varphi}\right]$$

$$\hat{\varphi}\frac{z\beta}{z^2+\beta^2} = \left[-\frac{\beta^n}{z}+\frac{z}{\beta}\hat{\varphi}\right] \rightarrow \hat{\varphi}\frac{z\beta}{z^2+\beta^2}-\frac{z}{\beta}\hat{\varphi} = -\frac{\beta^n}{z}$$

$$\left[\frac{z\beta}{z^2+\beta^2}-\frac{z}{\beta}\right]\hat{\varphi} = -\frac{\beta^n}{z} \rightarrow \frac{z\beta^2-z(z^2+\beta^2)}{\beta(z^2+\beta^2)}\hat{\varphi} = -\frac{\beta^n}{z}$$

$$\frac{-z^3}{\beta(z^2+\beta^2)}\hat{\varphi} = -\frac{\beta^n}{z} \rightarrow \hat{\varphi} = \frac{\frac{\beta^n}{z}}{\frac{z^3}{\beta(z^2+\beta^2)}} = \frac{\beta^{n+1}(z^2+\beta^2)}{z^4}\mathrm{conf}\left(\frac{1}{\beta}\right)$$

$$\hat{\varphi} = \frac{\left(z^2+\frac{1}{\beta^2}\right)}{\beta^{n+1}z^4} = \frac{(z^2\beta^2+1)}{\beta^{n+3}z^4}(z\beta^n)$$

$$\hat{\varphi} = \frac{(z^2\beta^2+1)}{\beta^3 z^3} \rightarrow$$

$$= \; 1+\frac{t^2}{2}$$

Example 2

$$\int_0^t y'(\tau)y(t-\tau)\,d\tau = 24y^3y(0) = 0$$

$$\left(\frac{z}{\beta^n}\right)\left[-y(x,0)\frac{\beta^n}{z}+\frac{z}{\beta}\hat{\varphi}\right] = 24\left[\frac{6\beta^{n+4}}{z^5}\right]$$

$$\frac{z^2}{\beta^{n+1}}\hat{\varphi}^2 = \frac{144\beta^{n+4}}{z^5} \rightarrow \hat{\varphi}^2 = \frac{\frac{144\beta^{n+4}}{z^5}}{\frac{z^2}{\beta^{n+1}}} = \frac{144\beta^{2n+5}}{z^7}$$

$$\hat{\varphi}^2 = 144\frac{\beta^{2n+5}}{z^7} \rightarrow f\left(\frac{1}{\beta}\right) \rightarrow \hat{\varphi}^2 = 144\frac{1}{z^7\beta^{2n+5}}(z\beta^n)$$

$$\hat{\varphi}^2 = 144\frac{1}{z^6\beta^{n+5}}(z\beta^n) = \frac{144}{z^5\beta^5} = \sqrt{\frac{144}{s^5}} = \frac{16}{\sqrt{\pi}}t^{3/2}$$

Note: Here it is multiplied twice by $z\beta^n$.

Example 3

$$\int_0^t y''(\tau)y'(t-\tau)\,d\tau = y'(t)-y(t)y(0) = y'(t) = 0 \quad \mathrm{sol.}\, y(t) = 0$$

$$\left(\frac{z}{\beta^n}\right)\left[-\frac{\beta^n}{z}y'(x,0)-\frac{\beta^n}{\beta}y(x,0)+\frac{z^2}{\beta^2}\hat{\varphi}\right]\left[-y'(x,0)\frac{\beta^n}{z}+\frac{z}{\beta}\hat{\varphi}\right] =$$

$$= \left[-y'(x,0)\frac{\beta^n}{z}+\frac{z}{\beta}\hat{\varphi}\right]-\hat{\varphi}$$

$$\left(\frac{z}{\beta^n}\right)\left(\frac{z^2}{\beta^2}\right)\widehat{\varphi}\left(\frac{z}{\beta}\widehat{\varphi}\right)=\frac{z}{\beta}\widehat{\varphi}-\widehat{\varphi}$$

$$\frac{z^4}{\beta^{n+4}}\widehat{\varphi}=\left[\frac{z}{\beta}-1\right]\widehat{\varphi}=0$$

$$\left(\frac{z^4}{\beta^{n+4}}-\left[\frac{z}{\beta}-1\right]\right)\widehat{\varphi}=0$$

$$\therefore y(t)=0$$

Conclusions

With these examples we have put into practice the use of the ZJ transform which is another way to solve these integral equations in their varieties that were presented, hoping that this will be supportive and helpful as another alternative method.

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