

## COMPLEX TRUSS ANALOGY USING PLASTIC AND ELASTIC ANALYSIS

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**ABSTRACT:** The work exposes the fundamentals of the plastic and elastic theory of analysis and degree, in order to sensitize its use. This work also compares the use of plastic method of structural analysis and elastic method of structural analysis using a complex truss system being acted upon by mobile load as a case study. This project disputes the common fact that only the elastic method of analysis is used to analyse mobile loads on truss systems by introducing stipulated steps in plastic method of analysis for analyzing truss systems carrying mobile loads. This project deals with the creation of a computer application that analyzes and designs structural trusses. This program was created using the relatively new C# programming language. The project also discusses various theoretical analysis techniques that can be implemented in developing a computer program. The main theoretical methods used in this project are influence line analysis and plastic method of analysis of mobile loads. The Reinforced concrete design is based on the EC3 code. The project solved the reactions on each member using the influence line analysis for truss system by taking cognizance of the position of the mobile load (at position  $x = 2a$ ), at a particular time. The complex truss system was analysed using the plastic method by unpinning the members of the truss system to form a frame and beams, for easy analysis. This project designed the complex system using the current code for design regulation i.e. the Euro codes. The bracing members were analysed and designed as a beam, fixed at the both ends and also at the middle. The results obtained from the research of this work shows that the influence line analysis generates higher axial forces on members which include;  $F_{C-D} = \frac{W_2 8a^2}{2h}$  kN (compression),  $F_{C-I}$ , and  $F_{C-J} = \frac{W_2 a \sqrt{a^2 + h^2}}{h}$  kN both in compression and tension, then  $F_{C-J} = \frac{W_2 a}{3}$  kN (Tension) and  $\frac{4W_2 a}{3}$  kN (compression) and  $F_{J-I} = \frac{3W_2 a^2}{h}$  kN than the axial forces generated using the plastic analysis method which also include;  $J_y = \frac{W_3 a + M_p}{a}$  (kN) (maximum compressive axial force on vertical strut members),  $I_x = \frac{11M_p - 2W_3 a - 2W_4 h}{2h}$  kN tension and  $W_4$  kN tension for both the lower chord and the top chord respectively, when acted upon by the same magnitude of imposed live loads of  $W_2$  kN. This is so because the plastic method of analysis involves a lot of assumptions that makes it yet not advisable to be used in the analyses of trusses carrying mobile loads.

**keywords:** Analysis, Elasticity, Load, Plasticity, and system.

### Introduction:

Engineering is a professional art of applying science to the efficient conversion of natural resources for the benefit of man. Engineering therefore requires above all creative imagination to innovative useful application for natural phenomenon (Kamath and Reddy, 2011)

Basically there are two approaches to provide adequate strength of structures to support a given set of design loads: Elastic Design and Plastic Design. Drift checks are also required in actual design practice, but the focus of discussion herein will be limited to elastic and plastic method of designs on truss systems.

A truss is an assemblage of long, slender structural elements that are connected at their ends. Trusses find substantial use in modern construction, for instance as towers, bridges, scaffolding, etc. In addition to their practical importance as useful structures, truss elements have a dimensional simplicity that will help us extend further the concepts of mechanics introduced in the modules dealing with uniaxial response.

In recent years, the engineers have done a lot of work to know the behaviour of structures, when stressed beyond the elastic limit called plastic limit. This has led to the development of new theory popularly known as *plastic theory*.

With the recent increase in the development of software programs, research engineers have developed computer aided designs analogy using the available softwares on ground. Structures viz truss structures, concrete structures e.t.c. are designed using these developed software designs. These computer aided designs include using the methods of elastic and plastic analogy in the design of trusses.

**Aim of the work:** This work is aimed at determining the elastic and plastic analysis method of analysing complex truss systems carrying mobile loads and comparing the both analysis.

**Objectives of the study: The objectives of the work includes:**

1. The comparison between the plastic and elastic methods of analysis and design on truss systems.
2. The elastic cum plastic deformations of structures under mobile loading condition.
3. The various methods of analysing the internal stresses that occur in a structure under external mobile loading eg. Complex truss system.
4. The fundamental concepts of plastic analysis.
5. Understanding the basis of and limitations of plastic analysis approaches.

Literature Review: A truss is defined as a framework which gives a stable form capable of supporting considerable external load over a large span with the components parts stressed primarily in axial tension or compression (Ezeagu and Nwokoye, 2009). In a plane, a truss is composed of relatively slender members often forming triangular configurations (Mau, 2002).

A truss is one of the major types of engineering structures which provides a practical and economical solution for many engineering constructions, especially in the design of bridges and buildings that demand large spans (Ustundag, 2005).

Trusses are statically determinate when all the bar forces can be determined from the equations of statics alone. Otherwise the truss is statically indeterminate (Saouma, 2007).

Equilibrium is the most important concept of structural analysis. A structure that is initially at rest and remains at rest when acted upon by applied loads is said to be in a state of equilibrium (Shanmugam and Narayanan, 2008). The resultant of the external loads on the body and the supporting forces or reactions is zero. Engineering talk about two types of equilibrium; static and dynamic, although it can be argued that static equilibrium is a special case of dynamic equilibrium. Static equilibrium exists if all parts of a structure can be considered motionless. I.e. the structural parts, which are initially at rest, remain at rest when acted upon by a system of force, which therefore suggested that the combined resultant effect of the system of forces shall be neither a force nor a couple. Otherwise there will be a tendency for motion of the body.

When a structure is in equilibrium, every element or constituent part of it is also in equilibrium. This property is made use of in developing the concept of the free body diagram for elements of a structure (Buick and Graham, 2003). Compatibility is concerned with deformation. If compatibility is assumed then geometric fit is implied. That is, if a joint of structure moves, then the ends of the members connected to that joint move by the same amount, consistent with the nature of the connection. A solution is compatible if the displacement at all points is not a function of the path. Therefore, a displacement compatible solution involves the existence of a uniquely defined displacement field (Buick and Graham, 2003).

Compatibility conditions require that the displacements and rotations be continuous throughout the structure and compatible with the nature supports conditions. For example, at a fixed support this requires that displacement and slope should be zero (Kharagpur, 2012).

In the case of a pin-jointed frame, compatibility means that the ends of the member at a joint undergo equal translation. If the framework is rigidly joined, then, in addition to equal translation, the rotation of the ends of the members meeting at a joint must be equal. According to Okoro (2004) in a project work titled the plastic behaviour of structures (plastic analysis vs. elastic analysis) stated that the deformation of the structure set up strains and related internal stresses within the elements. Stress is related to strain through the stress-strain laws, which is a function of the type of material and the nature of the strain (Spencer, 1988). The best known stress-strain law is that which defines linear elastic behaviour. In this case, stress is proportional to strain and the constant of proportionality is Young's Modulus  $[E]$ . There are other stress-strain laws defining a wide range of behaviour (like the plastic behaviour) but it should be appreciated that all stress-strain laws are approximates. The major reason for interest in the assumption of linearity of structural behaviour is that it allows the use of principle of superposition. This principle means that the displacement resulting from each of a number of forces may be added to give the displacement resulting from the sum of the forces. Super-position also implies that the forces corresponding to a number of displacements may be added to yield the force corresponding to the sum of the displacements. The principle must not be used for analysis of non-linear structures or in the methods of plastic theory.

Elastic materials are such that returns to its original state after undergoing an extension by an external force once its elastic limit is not exceeded.

In the analysis of these two behaviour viz: elastic and plastic, there are differences in the mode or method of analysis: In plastic analysis, the collapse load and load factor used in the design of such structure plays a vital role. Once these two criteria are adequately catered for in the design, the structure would withstand any applied force with visible deformations while in the elastic analysis, deflection limit is the major criteria for design. It is design such that it could remain functional under a certain applied force once the deflection limit is not exceeded.

The plastic method can be seen as a more rational method for design because all parts of the structure can be given the same safety factor against collapse. In contrast for elastic methods the safety factor varies. Intrinsically the plastic method of analysis is simpler than the elastic method because there is no need to satisfy elastic strain

compatibility conditions. However calculations for instability and elastic deflections require careful consideration when using the plastic method, but nevertheless it is very popular for the design of some structures (e.g. beams and portal frames) (Martin and Purkiss, 2008).

Deflection becomes the governing factor of elastic analysis while the collapse load and load factor remains basis of plastic analysis. If these criteria are strictly adhere to in the design of a structure using either of the analysis method, then the success of the structure to a larger extent is certain. The plastic and elastic method of analysis has various areas where they are applicable depending on the designer's choice and conservation in terms of material. Since the plastic method of analysis is more economical in terms of material than the elastic method of analysis.

The plastic method of analysis is used especially in the design of steel structure e.g. rigid frames, indeterminate rigid frames. Since collapse load is the major criterion that which plastic method is based on, it is adequately taken care of in the use of this design, thus design of ductile structure is plastic in nature so, plastic design is basically needed.

It does not mean that elastic method is not also applicable in steel structure, but it is restricted to some extent. Elastic method is used mainly in the design of reinforced concrete. The ability of a concrete structure to maintain its shape or form under given deformation or deflection makes it elastic in nature as it would without some degree of deformations and when exceeded, cracks makes this methods suitable for its use.

According to Okoro (2004) in a project work titled the plastic behaviour of structures (plastic analysis vs. elastic analysis) stated that the traditional method of showing a typical engineering problem prior to the introduction of the electronic digital computer was initially by a mathematician or an engineer who endeavoured to obtain a solution based on strict scientific reasoning and with no regard to the resulting calculation (Litton, 1973). However, this process was terminated by the introduction of the electronic computer, which provided the opportunity for a fresh approach to the problem.

**Theory of plasticity :** The theory of plasticity is the branch of mechanics that deals with the calculation of stresses and strains in a body, made of ductile material, permanently deformed by a set of applied forces (Chakrabarty, 2006). The theory is based on certain experimental observations on the macroscopic behaviour of metals in uniform states of combined stresses. The observed results are then idealized into a mathematical formulation to describe the behaviour of metals under complex stresses.

Unlike elastic solids, in which the state of strain depends only on the final state of stress, the deformation that occurs in a plastic solid is determined by the complete history of the loading. The plasticity problem is, therefore, essentially incremental in nature, the final distortion of the solid being obtained as the sum total of the incremental distortions following the strain path.

## METHODOLOGY

**Development of elastic model** Consider the bridge in Fig. 3.0. As the car moves across the bridge, the forces in the truss members change with the position of the car and the maximum force in each member will be at a different car location. The design of each member must be based on the maximum probable load each member will experience.

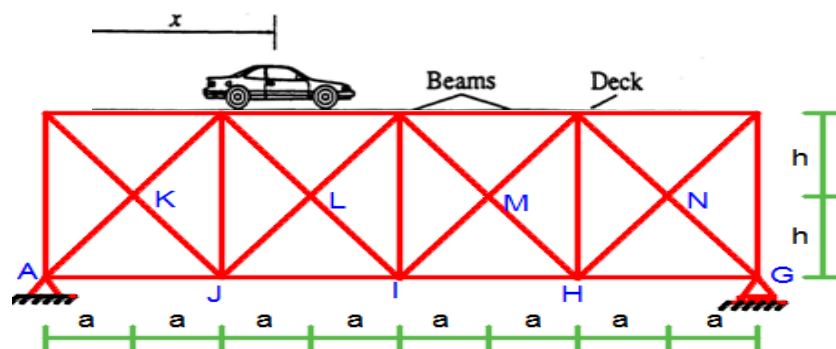


Fig. 3.1: Mobile load on truss system

Therefore, the truss analysis for each member would involve determining the load position that causes the greatest force or stress in each member.

If a structure is to be safely designed, members must be proportioned such that the maximum force produced by dead and live loads is less than the available section capacity.

Structural analysis for variable loads consists of two steps:

- a. Determining the positions of the loads at which the response function is maximum; and

b. Computing the maximum value of the response function.

Once an influence line is constructed;

- Determine where to place live load on a structure to maximize the drawn response function; *and*
- Evaluate the maximum magnitude of the response function based on the loading.

Consider a through type composite truss of 8 panels, each of length “a” and height “2h” as shown in Fig. 3.0. A little consideration will show, that the truss consists of;

- (i) A primary truss of panels each length of  $2a$  and height  $2h$ . (i.e. with members as CD,JI,CJ,CI and DJ)
- (ii) 8 secondary trusses of 1 panel, each of length  $2a$  and of height  $h$  as shown in Fig. 3.0

A little consideration will show, that some of the members in the second panel of the primary truss occur in the primary truss only (e.g. CD, JI, CJ, and DI). Some members occur in both the primary as well as secondary trusses (e.g. CL, LI, DL and LI). The influence lines for the members, which occur in the primary truss only, will be given by the influence line for the corresponding member of the primary truss only. But the influence lines for the members, which occur in both the primary and secondary trusses, may be drawn by joining the points obtained by algebraically adding the ordinates of the influence lines for the corresponding members in the primary as well as secondary truss. Cut a section on the member to be analyzed before analyzing.

The maximum force on any member due to a particular moving load is gotten by multiplying the moving load with the area of the influence line diagram.

**Development of plastic model :** Consider the truss shown in Fig. 3.0 above, as the load moves on the truss it distributes a udl on the top cord of the truss system.

Analysis of the complicated truss system as shown above using plastic method of analysis is rather not easy. Therefore, for easier analysis, the members of the complicated truss system were unpinned into frames, beams and bracing members. The top chord members and the struts (vertical members) were joined together to form a continuous frame with uniformly distributed loading of  $W_3$  kN. The frame is pinned at the end reactions and fixed at the middle reactions. The internal members are analyzed as a continuous beam with uniformly distributed loading of  $W_6$  kN. The continuous beam is pinned at both ends and also at the middle. The lower cord members were analyzed as a simple beam with uniformly distributed loading of  $W_7$  kN, pinned at one end and having roller support at the other end.

**N.B** – The ratio of the corresponding plastic section modulus ( $Z_p$ ) to the corresponding elastic section modulus ( $Z_e$ ) of a particular section (both gotten from table) gives the Shape factor (S) of the section. The shape factor is being multiplied by the factor of safety of the section (1.5) to give the section’s load factor ( $L_F$ ). The Load factor multiplied by the corresponding loading on each member gives the total load to be used for the plastic analysis of the member.

Combined mechanism - The independent mechanisms are combined to determine the maximum  $M_p$  value required to induce collapse with the minimum number of hinges. The shear forces at the various reactions were obtained. For plastic analysis, the axial forces acting on each member are used for the design of the member and it also shows whether the member is being acted upon by compression or tension forces.

**Design of truss system :** The top chord members are designed as I or H steel beam section. The vertical members (struts) were designed as T or equal long angles back to back steel column section. The internal bracing members were designed as L- steel bracing section. In the structural design of steel structures, reference to standard code is essential. As EC3 will eventually replace BS 5950 as the new code of practice, it is necessary to study and understand the concept of design methods in EC3.

Codes of practice provide detailed guidance and recommendations on design of structural elements. Buckling resistance and shear resistance are two major elements of structural steel design. Therefore, provision for these topics is covered in certain sections of the codes. The study on Eurocode 3 in this project will focus on the subject of moment and shear design.

### **Design of Steel Beam According to EC3**

The design of simply supported steel beam covers all the elements stated below. Sectional size chosen should satisfy the criteria as stated below:

- (i) Cross-sectional classification
- (ii) Shear capacity
- (iii) Moment capacity (Low shear or High shear)
- (iv) Bearing capacity of web
  - a) Crushing resistance
  - b) Crippling resistance
  - c) Buckling resistance
- (v) Deflection

## COMPLEX TRUSS ANALOGY USING PLASTIC AND ELASTIC ANALYSIS

Analysis, design and comparison works will follow subsequently. Beams and columns are designed for the maximum moment and shear force obtained from computer software analysis. Checking on several elements, such as shear capacity, moment capacity, bearing capacity, buckling capacity and deflection is carried out. Next, analysis on the difference between the results using the two analysis (Elastic and Plastic) is done. Eventually, comparison of the results will lead to recognizing the difference in design approach for each analysis.

**Load distribution : Top Chord members** – The top cord members carry a uniformly distributed loading of say  $W_3\text{kN}$ , which comprises of the total live load acting on the member and the member self weight (dead load). Both multiplied by their factors of safety.

**Vertical members (struts)** – The vertical members carry a loading of say  $W_8\text{kN}$ , which comprises of the total loading coming from the top cord and the wind load multiplied by its factor of safety.

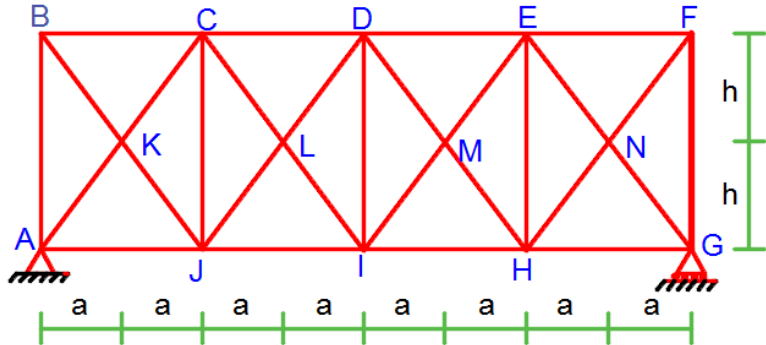
**Bracing members** – The bracing members are being acted upon by wind loads, their own self weight and----- multiplied by their various factors of safety which is denoted as  $W_9\text{kN}$ .

**The lower chord** – The lower chord members are being acted upon by the live load and the self-weight of the whole truss system multiplied by their various factors of safety, denoted in the project as  $W_{10}\text{kN}$ .

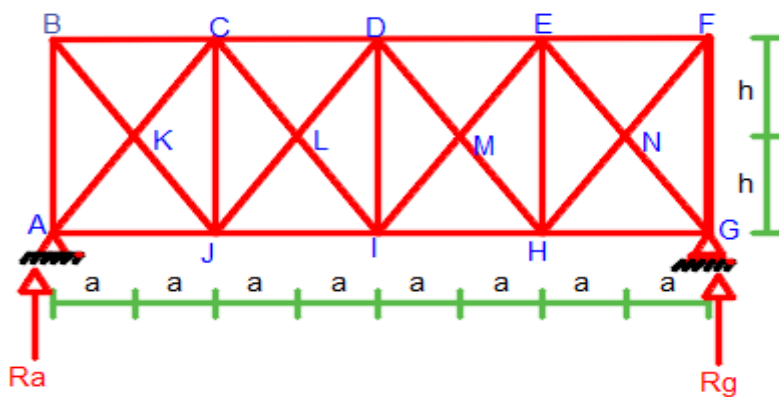
**Computer application of plastic and elastic analysis of truss system** : For the purpose of this project, we are going to analyze the complicated truss system using the C# programming language. The elastic and plastic analysis of complicated truss system discussed in chapter four were encoded in the C# programming language for easier analysis and to meet up with the growing trend of technology in the modern world.

**C#** (pronounced "see sharp") is a computer programming language. It is developed by Microsoft. It was created to use all capacities of .NET platform. The first version was released in 2001. The most recent version is C# 5.0, which was released in August 2012. C# is a modern language

### TRUSS ANALYSIS

Member ref.	Calculation	Output
	<p><b>Design data:</b>  Span of truss (L) = <math>8a(\text{m})</math>  Height of truss = <math>2h(\text{m})</math>  Bracing length = <math>2\sqrt{h^2 + a^2}</math> (m)  Span of each stanchion = <math>2a(\text{m})</math>  Let position of the load on any member at a particular time be = <math>x</math>  Bracing slope (<math>\Theta</math>) = <math>\tan^{-1}\left(\frac{2h}{2a}\right)</math>  Let the moving load longer than the span be <math>W_2(\text{kN/m})</math></p>  <p><b>Fig. 4.0: Truss system</b></p>	

#### 4.1 INFLUENCE LINE ANALYSIS (Unit Load)



**Fig. 4.1:** Truss system with reactions

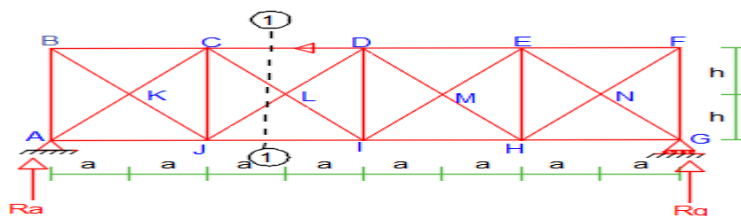
Let the unit load be  $w(\text{kN})$

##### For Reactions

$$R_a = \frac{(L-x)w}{L} = \frac{(8a-4a)}{8a} \times 1(\text{kN})$$

$$R_g = \frac{(x)w}{L} = \frac{(4a) \times 1}{8a} (\text{kN})$$

##### Influence line for force in member C-D



**Fig. 4.2**

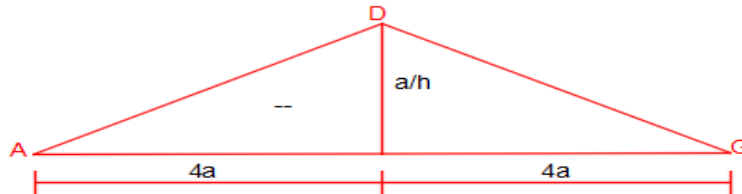
Passing section 1-1 cutting the member  $C-D$  as shown in **fig. 3** above. Member  $C-D$  occurs in the primary truss only, therefore the influence line for this member may be drawn from the primary truss only.



Influence line for C-D =  $\frac{\text{Bending Moment for influence line for } I}{\text{Vertical distance between D-I}}$

Influence line (I.L) for Bending moment (B.M) at I is a triangle with ordinate =  $\frac{x(L-x)}{L} = \frac{4a(8a-4a)}{8a} = 2a \text{ (kN)}$

Therefore I.L for force in member C-D will also be a triangle with ordinate =  $2a \times \frac{1}{2h} = \frac{a}{h} \text{ (kN)}$



**Fig 4.3:** Influence line diagram for member C-D

$$F_{C-D} = W_2 \times \text{Area of ADG}$$

$$= W_2 \times \frac{1}{2} \times 8a \times \frac{a}{h} \text{ (compression)}$$

**Influence line for force in member C-I**

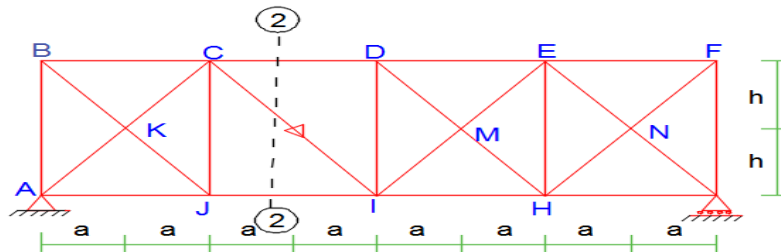
Member C-I occurs both in primary and secondary trusses. And as such will be calculated as stated in **chapter 3**.

**For Primary truss**

Passing a section 2-2 cutting the member C-I as shown in fig. 4.5 below.

$$F_{C-D} = \frac{W_2 8a^2}{2h}$$

(compression)



**Fig. 4.4**

I.L for member C-I b/w AJ:  $F_{CI} \sin \theta = R_G$

$$F_{CI} = \frac{R_G}{\sin \theta} = \frac{x \csc \theta}{L}$$

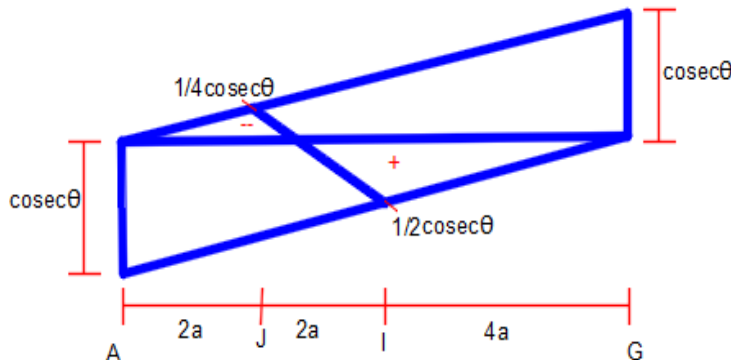
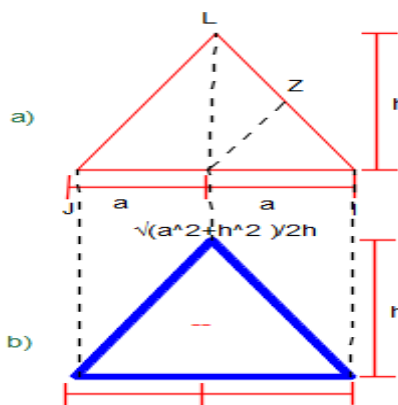
Where:  $x = 2a$  and  $L = 8a$

$$I.L_{CI} = \frac{2a \csc \theta}{8a} = \frac{\csc \theta}{4} \text{ (kN)}$$

I.L for member C-I b/w:  $F_{CI} \sin \theta = R_A$

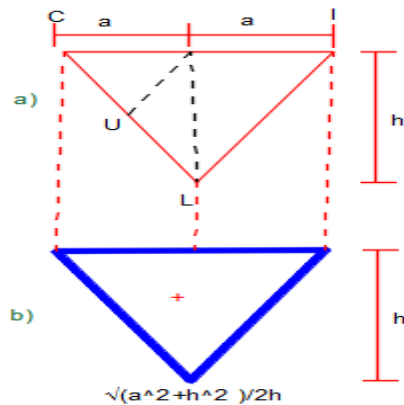
$$F_{C-I} = \frac{\csc \theta}{4} \text{ (kN)}$$

(compression)

	$F_{CI} = \frac{R_A}{\sin \theta} = \frac{(L - x/L)}{\sin \theta}$ <p>Where: <math>x = 4a</math> and <math>L = 8a</math></p> $F_{CI} = \frac{(8a - 4a/8a)}{\sin \theta} = \frac{1}{2} \operatorname{cosec} \theta (kN)$ <p>The I.L between the joints J and I will be a straight line joining the ordinate under the joints J and I as shown below.</p>  <p><b>Fig. 4.5:</b> Influence line diagram of the primary truss</p>	$F_{C-I} = \frac{1}{2} \operatorname{cosec} \theta (kN)$ <p>(Tension)</p>
	<p><u>For the secondary truss</u></p> <p><u>I.L for force in member L – I:</u></p>  <p><b>Fig. 4.6:</b> Influence line diagram of the secondary truss</p> <p>The I.L for force in member L-I in the secondary truss will be given by the I.L for bending moment at Q (the mid-point between points J and I) opposite joint L divided by the vertical distance between the member L-I and the opposite joint distance Q as shown in fig. 4.7b above.</p> <p>I.L for B.M at Q is a triangle with ordinate <math>= \frac{a \times a}{2a} = \frac{a}{2}</math></p> <p>Therefore, I.L for member L.I <math>= \frac{a}{2} \times \frac{1}{ZQ}</math></p> $= \frac{a}{2} \times \frac{1}{ah/\sqrt{a^2+h^2}} = \frac{\sqrt{a^2+h^2}}{2h} (kN) \text{ (Compression)}$ <p><u>I.L for force in member L – I:</u></p>	$I.L_{L-I} = \frac{\sqrt{a^2+h^2}}{2h} (kN)$ <p>(Compression)</p>



The I.L for force in member C-L in the secondary truss will be given by the I.L for bending moment at the opposite joint Y divided by /UY/ i.e the vertical distance between the member CL and the opposite joint Y as shown in fig. 4.4 above.

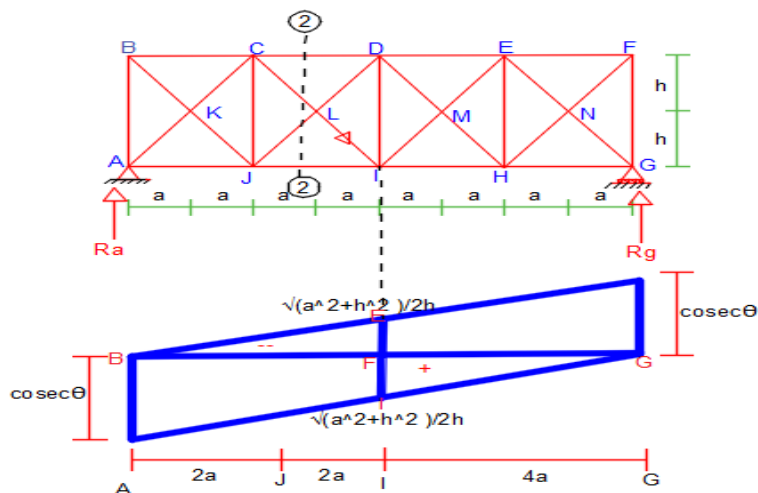


**Fig. 4.7:** Influence line diagram of the secondary truss

I.L for B.M at Y is a triangle with ordinate  $= \frac{a \times a}{2a} = \frac{a}{2}$

Therefore, I.L for member C.L  $= \frac{a}{2} \times \frac{1}{UY}$   
 $= \frac{a}{2} \times \frac{1}{ah / \sqrt{a^2 + h^2}} = \frac{\sqrt{a^2 + h^2}}{2h}$  (kN) (Tension)

Superposing influence lines C-I, L-I and C-L we will have;



**Fig. 4.8:** Influence line diagram of member C - I

$$I.L_{C.L} = \frac{\sqrt{a^2 + h^2}}{2h} \text{ (kN)} \text{ (Tension)}$$

$$F_{Max} = W_2 \times \text{Area of BEF}$$

$$= W_2 \times \frac{1}{2} \times 4a \times \frac{\sqrt{a^2 + h^2}}{2h} \text{ (compression)}$$

$$F_{Max} = W_2 \times \text{Area of FGI}$$

$$= W_2 \times \frac{1}{2} \times 4a \times \frac{\sqrt{a^2 + h^2}}{2h} \text{ (Tension)}$$

$$F_{Max} = \frac{W_2 a \sqrt{a^2 + h^2}}{h} \text{ (kN)}$$

(compression)

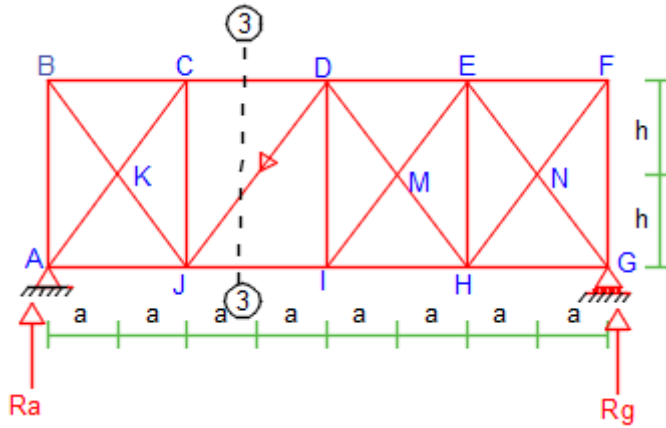
$$F_{Max} =$$

*Influence line for force in member D-J*

Member D – J occurs both in primary and secondary trusses. And as such will be calculated as stated in **chapter 3**.

*For Primary truss*

Passing a section 2 – 2 cutting the member D – J as shown in fig. 4.5 below.



**Fig. 4.9**

I.L for member D – J b/w /BC/:  $I.L_{DJ} \sin \theta = R_G$

$$I.L_{DJ} = \frac{R_G}{\sin \theta} = \frac{x \csc \theta}{L}$$

Where:  $x = 2a$  and  $L = 8a$

$$I.L_{DJ} = \frac{2a \operatorname{cosec} \theta}{8a} = \frac{\operatorname{cosec} \theta}{4} \text{ (kN)}$$

$$\frac{W_2 a \sqrt{a^2 + h^2}}{h} \text{ (kN)}$$

(Tension)

$$F_{D-J} = \frac{\text{cosec}\theta}{4} \text{ (kN)}$$

(Tension)

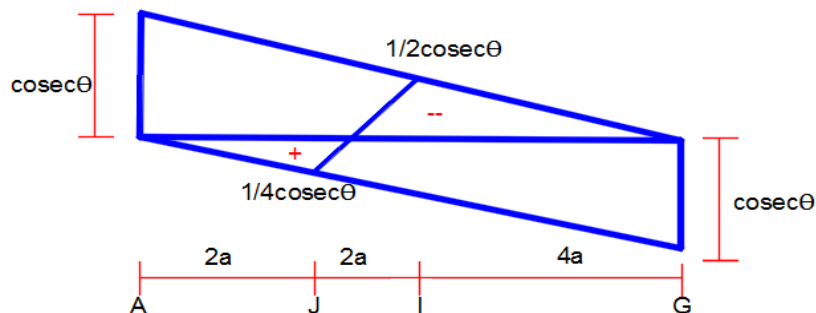
$$\underline{\text{I.L for member C - I b/w /DF/ : } I . L_{DJ} \sin \theta = R_A}$$

$$I.L_{DJ} = \frac{R_A}{\sin \theta} = \frac{(L - x/L)}{\sin \theta}$$

Where:  $x = 4a$  and  $L = 8a$

$$F_{CI} = \frac{(8a - 4a/8a)}{\sin \theta} = \frac{1}{2} \operatorname{cosec} \theta (kN)$$

The I.L between the joints C and D will be a straight line joining the ordinate under the joints C and D as shown below.



**Fig. 4.10:** Influence line diagram of the primary truss

$$\frac{F_{D-J}}{2} = \frac{\operatorname{cosec} \theta}{2} (kN)$$

(compression)

**For the secondary truss**

I.L for force in member L – J:

Member L-J has the same I.L diagram with member L-I

$$= \frac{\sqrt{a^2+h^2}}{2h} \text{ (kN)(compression)}$$

I.L for force in member D – L:

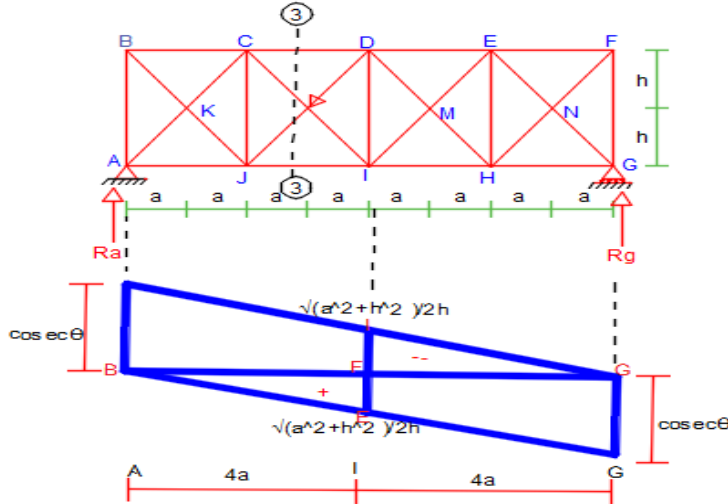
Member D-L has the same I.L diagram with member C-L

$$= \frac{\sqrt{a^2+h^2}}{2h} \text{ (kN)(Tension)}$$

$$\text{I.L}_{\text{C.L}} = \frac{\sqrt{a^2+h^2}}{2h} \text{ (kN)} \\ \text{(compression)}$$

$$\text{I.L}_{\text{C.L}} = \frac{\sqrt{a^2+h^2}}{2h} \text{ (kN)} \\ \text{(Tension)}$$

Superposing influence lines D - J, D - L and L - J we will have;



**Fig. 4.11: Influence line diagram of member D - J**

$$F_{\text{Max}} = W_2 \times \text{Area of FGI}$$

$$= W_2 \times \frac{1}{2} \times 4a \times \frac{\sqrt{a^2+h^2}}{2h} \text{ (compression)}$$

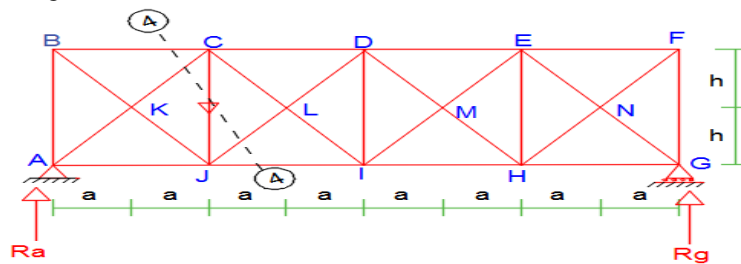
$$F_{\text{Max}} = W_2 \times \text{Area of BEF}$$

$$= W_2 \times \frac{1}{2} \times 4a \times \frac{\sqrt{a^2+h^2}}{2h} \text{ (Tension)}$$

$$F_{\text{Max}} = \frac{W_2 a \sqrt{a^2+h^2}}{h} \text{ (kN)} \\ \text{(compression)}$$

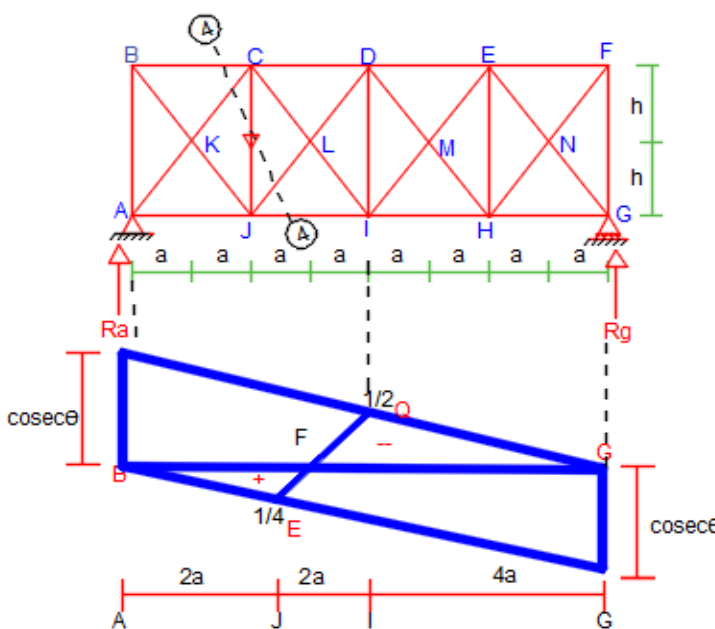
**Influence line for force in member C-J**

Cutting the section 4 – 4 as shown below;



**Fig. 4.12**

$$F_{\text{Max}} = \frac{W_2 a \sqrt{a^2+h^2}}{h} \text{ (kN)} \\ \text{(Tension)}$$

	<p><u>I.L for member C – J between /AJ/:</u>  Since the force <math>R_G</math> acts upwards, force in member C-J will be acting downwards thus causing tension.  <math display="block">I.L_{A-J} = R_G = \frac{x}{L}</math> Where; <math>x = 2a</math> and <math>L = 8a</math>  <math display="block">I.L_{A-J} = \frac{2a}{8a} = \frac{1}{4} \text{ (Tension)}</math></p> <p><u>I.L for member C – J between /IG/:</u>  Considering the left;  <math display="block">I.L_{A-J} = R_A = \frac{(L-x)}{L}</math> Where; <math>x = 4a</math> and <math>L = 8a</math>  <math display="block">I.L_{A-J} = \frac{4a}{8a} = \frac{1}{2} \text{ (compression)}</math></p>  <p><b>Fig. 4.13: Influence line diagram of member C - J</b></p>	<p><math display="block">I.L_{A-J} = \frac{1}{4}</math>  (Tension)</p> <p><math display="block">I.L_{A-J} = \frac{1}{2}</math>  (compression)</p>
	<p> <math display="block">F_{Max} = W_2 \times \text{Area of FQG}</math> <math display="block">= W_2 \times \frac{1}{2} \times \left(4a + \frac{4a}{3}\right) \times \frac{1}{2} \text{ (compression)}</math> <math display="block">F_{Max} = W_2 \times \text{Area of BEF}</math> <math display="block">= W_2 \times \frac{1}{2} \times \left(2a + \frac{2a}{3}\right) \times \frac{1}{4} \text{ (Tension)}</math> <p style="text-align: center;"><b><u>Influence line for force in member J-I</u></b></p> </p>	<p> <math display="block">F_{Max} = \frac{4W_2a}{3}</math>  (compression) </p> <p> <math display="block">F_{Max} = \frac{W_2a}{3}</math>  (Tension) </p>

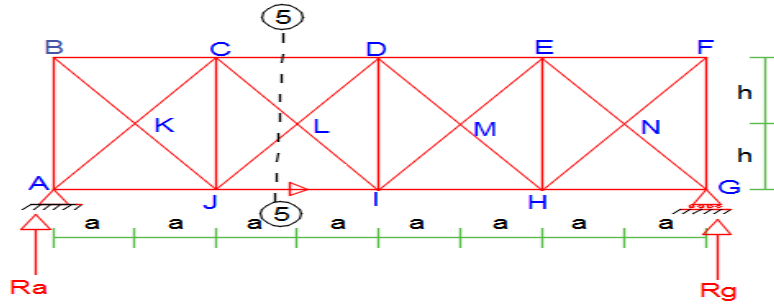


Fig. 4.14

Cutting the section 5 – 5 as shown above;  
I.L for force in member J-I will be given by the I.L for bending moment at the opposite joint C (triangle) divided by the vertical distance between the member J-I and the opposite joint C.

$$\text{I.L at joint C} = \frac{x(L-x)}{L}$$

$$\text{I.L at member H-G} = \frac{x(L-x)/L}{2h}$$

Where;  $x = 2a$  and  $L = 8a$

$$\text{I.L at member H-G} = \frac{2a(8a-2a)/8a}{2h} = \frac{3a}{4h} \text{ (Tension)}$$

$$\text{I.L}_{H-G} = \frac{3a}{4h} \text{ (Tension)}$$

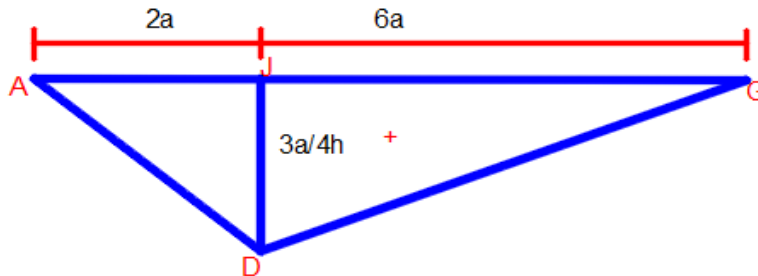


Fig. 4.15: Influence line diagram of member J - I

$$\begin{aligned} F_{\text{Max}} &= W_2 \times \text{Area of ADG} \\ &= W_2 \times \frac{1}{2} \times 8a \times \frac{3a}{4h} \text{ (Tension)} \end{aligned}$$

$$F_{\text{Max}} = \frac{3W_2a^2}{h} \text{ (Tension)}$$

#### FORCES IN THE TRUSS MEMBERS

Member	Tension (kN)	Compression (kN)
$F_{C-D}$		$\frac{W_2 8a^2}{2h}$

$F_{C-I}$	$\frac{W_2 a \sqrt{a^2 + h^2}}{h}$	$\frac{W_2 a \sqrt{a^2 + h^2}}{h}$	
$F_{D-J}$	$\frac{W_2 a \sqrt{a^2 + h^2}}{h}$	$\frac{W_2 a \sqrt{a^2 + h^2}}{h}$	
$F_{C-J}$	$\frac{W_2 a}{3}$	$\frac{4W_2 a}{3}$	
$F_{I-I}$	$\frac{3W_2 a^2}{h}$		

Table 1

#### 4.2 PLASTIC MOMENT ANALYSIS

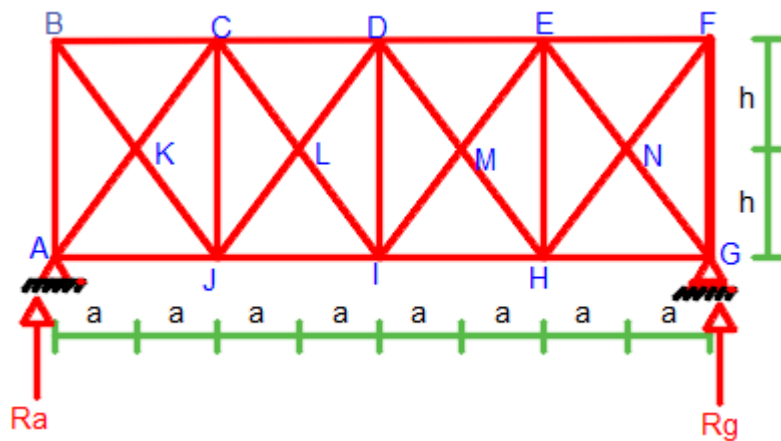


Fig. 4.16

No of degree of indeterminacy( $I_D$ ) =  $(m + r) - 2n$

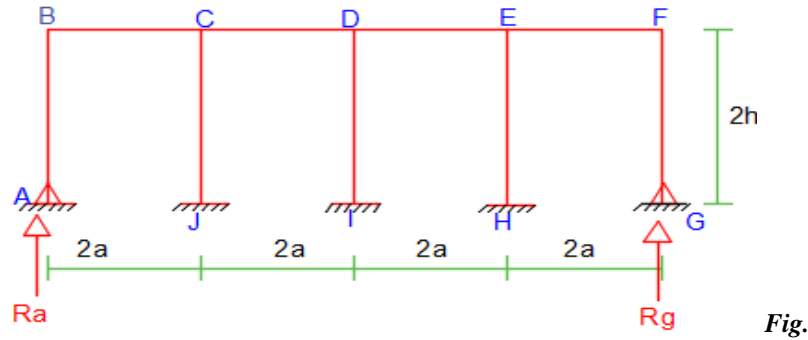
Where; **m** (number of members) = 21

**r**(number of reactions) = 3

**n**(number of nodal points) = 10

$(I_D) = (21 + 3) - (2 \times 10) = 4$

Unpinning the lower members and the bracing members from the truss system leaving the frame structure for easy analysis we have;



4.17

No of degree of indeterminacy( $I_D$ ) =  $(3m + r) - 3n$

Where;  $m$  (number of members) = 9

$r$ (number of reactions) = 10

$n$ (number of nodal points) = 13

$(I_D) = ((3 \times 9) + 13) - (3 \times 10) = 10$

No of possible position of hinges = 18

@ Joints B, B<sup>1</sup>, C(1,2,3), C<sup>1</sup>, D(1,2,3), D<sup>1</sup>, E(1,2,3), E<sup>1</sup>, F, H, I and J.

No of independent collapse mechanism =  $18 - 10 = 8$

4 Beam mechanisms – Beam B – C

– Beam C – D

– Beam D – E

– Beam E – F

ii) 3 Joint mechanisms – Joint C(1,2,3)

– Joint D(1,2,3)

– Joint E(1,2,3)

iii) 1 sway mechanism

**Collapse mechanism 1 – BEAM C – D**

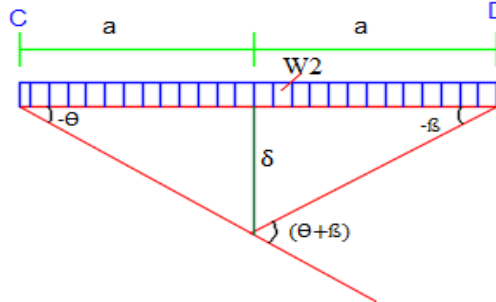


Fig. 4.18

$W_2$  = load factor  $\times$  the imposed loading

$\theta = a\theta = a\beta$

$\therefore \theta = \beta$

Internal work done (I.W) =  $Mp(\theta) + Mp(\beta) + Mp(\theta + \beta)$   
 $= 4Mp\theta$

External work done (E.W) =  $W_2 \times 2a \times \frac{\theta}{2}$   
 $= a^2 W_2 \theta (kN\theta m)$

Considering work equation: Internal work done = External work done

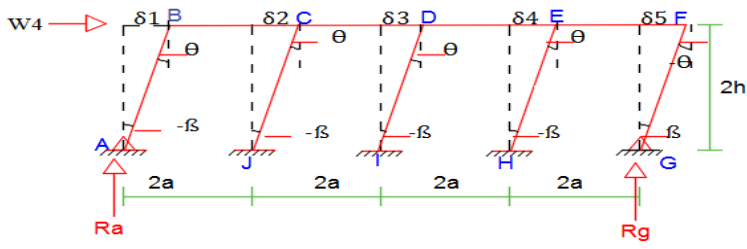
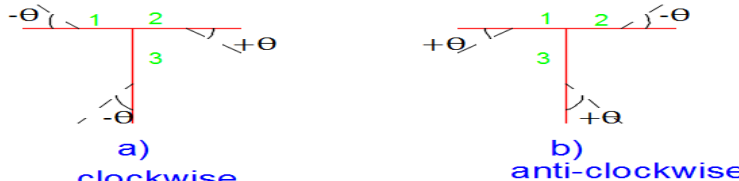
$4Mp\theta = a^2 W_2 \theta (kN\theta m)$

$\therefore Mp = \frac{a^2 W_2}{4} (kNm)$

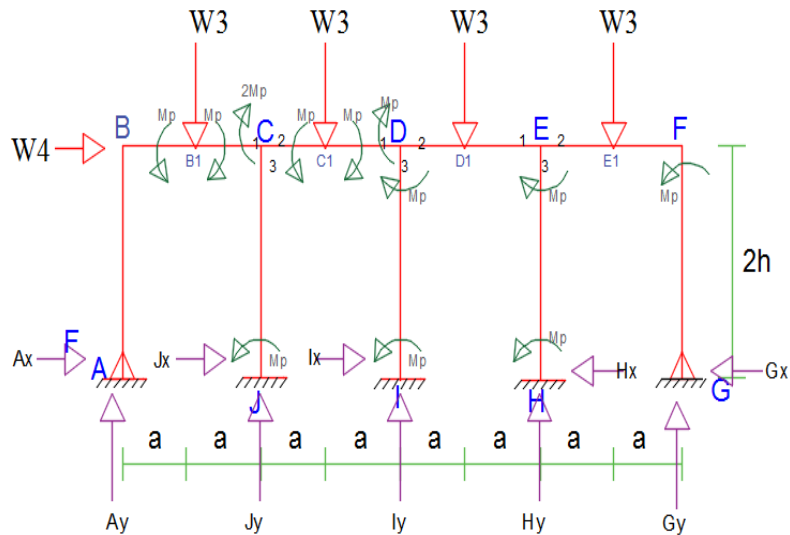
Other beam mechanisms (Beams; B – C, D – E and E – F) have the same methodology and the same  $Mp$  with Beam mechanism B – C above.

$$Mp = \frac{a^2 W_2}{4} (kNm)$$



	<p style="text-align: center;"><b><u>Collapse mechanism 2 – SWAY MECHANISM</u></b></p>  <p><b>Fig. 4.19</b>  <math>\partial_1 = \partial_2 = \partial_3 = \partial_4 = \partial_5 = 2h\theta</math>  <math>\therefore \theta = \beta</math>              Internal work done (I.W)  <math>= Mp(\theta) + Mp(\theta) + Mp(\beta) + Mp(\theta) + Mp(\beta) + Mp(\theta) + Mp(\beta)</math>  <math>= 8Mp\theta</math>              External work done (E.W) = <math>W_4 \times \partial_2</math>  <math>= 2h\theta W_4 \text{ (kNm)}</math>              Considering work equation: Internal work done = External work done  <math>8Mp\theta = 2h\theta W_4 \text{ (kNm)}</math>  <math>\therefore Mp = \frac{W_4 h}{4} \text{ (kNm)}</math></p> <p style="text-align: center;"><b><u>Collapse mechanism 3 – JOINT MECHANISM {Joint C (1,2,3)}</u></b></p>  <p><b>Fig. 4.20</b></p> <p style="text-align: center;">Considering clockwise rotation in the combine mechanism              1 – (+<math>\theta</math>)              2 – (-<math>\theta</math>)              3 – (+<math>\theta</math>)</p> <p style="text-align: center;"><b><u>Combined mechanism</u></b></p> <p>The independent mechanisms are combined to determine the maximum <math>M_p</math> value required to induce collapse with the minimum number of hinges.              In this case the following combinations have been evaluated;</p>	<p>I.W = <math>8Mp\theta</math></p> <p>E.W  <math>= 2h\theta W_4 \text{ (kNm)}</math></p> <p><math>Mp</math>  <math>= \frac{W_4 h}{4} \text{ (kNm)}</math></p>

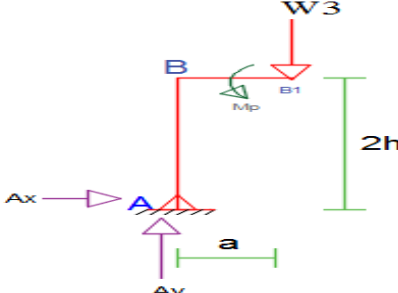
Member ref.	Calculation	Output
	<p style="text-align: center;"><b><u>DETERMINATION OF THE REACTIONS</u></b></p> <p>Checking collapse mechanism XIV with hinges at B<sup>1</sup>, C<sub>1</sub>, C<sup>1</sup>, D<sub>4</sub>, D<sub>6</sub>, E<sub>9</sub>, F, H, I and J (i.e 10 hinges). The value of the <math>M_p</math> obtained <math>\left( \frac{2(a^2 W_2) + 2h W_4}{13} \text{ kNm} \right)</math> should be checked by ensuring that the bending moment in the frame does not exceed the relevant <math>M_p</math> value at any location.</p>	

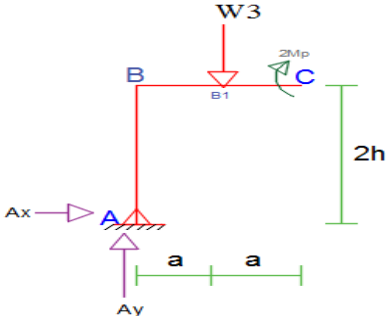
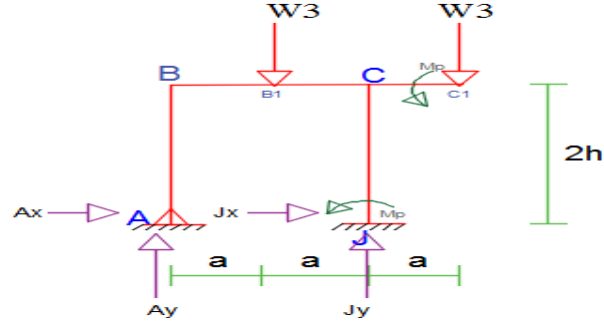


**Fig. 4.21:** Position of  $M_p$  on the continuous frame structure

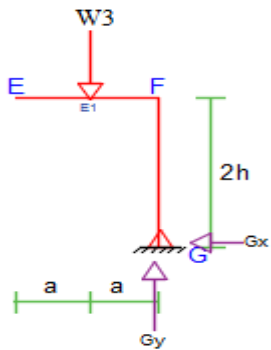
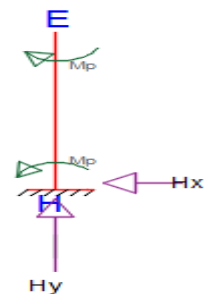
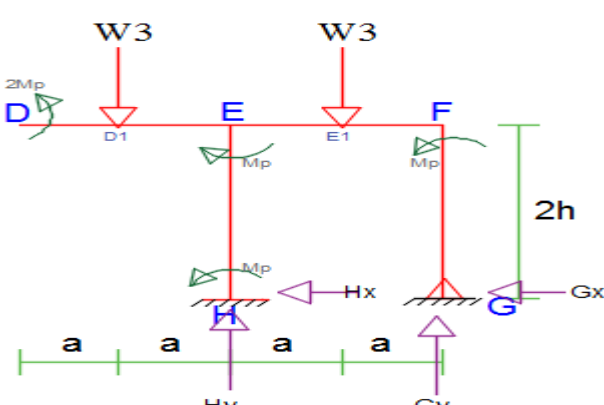
Here  $W_3$  (kN) = UDL ( $W_2$  kN)  $\times$  bay width ( $2a$  m)

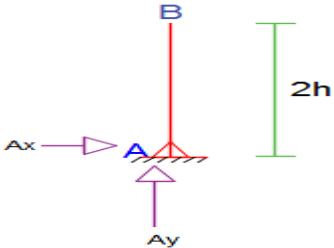
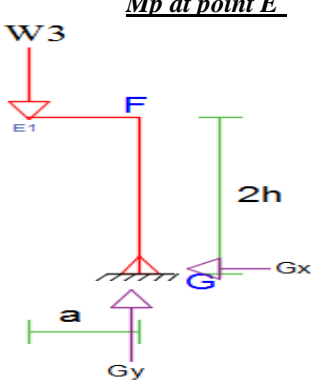
Consider the equilibrium of the left-hand side of the frame at  $B^1$  and at joint  $C_1$ .

Member ref.	Calculation	Output
	 <p style="text-align: center;"><b>Fig. 4.22</b></p> $\sum M_{B1} = 0$ $\therefore (A_y \times a) - (A_x \times 2h) - M_p = 0$ $\therefore aA_y - 2hA_x = M_p \text{ ----- } \{1\}$	

	 <p style="text-align: center;"><b>Fig. 4.23</b></p> <p> <math>\sum M_{B1} = 0</math>  <math>\therefore (A_y \times 2a) - (A_x \times 2h) - (W_3 \times a) + 2M_p = 0</math>  <math>\therefore 2aA_y - 2hA_x = W_3a - 2M_p \text{ ----- } \{2\}</math> </p> <p>From equation {1}; <math>A_y = \frac{M_p + 2hA_x}{a} \text{ ----- } \{3\}</math>  <i>Substituting the value of <math>A_y</math> in equation {3} into equation {2} we have;</i>  <math>A_x = \frac{W_3a - 4M_p}{2h} \text{ (kN) ----- } \{4\}</math> </p> <p><i>Substituting the value of <math>A_x</math> in equation {4} into equation {1} we have;</i>  <math>A_y = \frac{W_3a - 3M_p}{a} \text{ (kN) ----- } \{5\}</math> </p>	$A_x = \frac{W_3a - 4M_p}{2h} \text{ (kN)}$ $A_y = \frac{W_3a - 3M_p}{a} \text{ (kN)}$
<b>Member ref.</b>	<b>Calculation</b>	<b>Output</b>
	<p>Considering the equilibrium of the left-hand side of the frame at C<sup>1</sup> and at joint D<sub>1</sub>.</p>  <p style="text-align: center;"><b>Fig. 4.24</b></p> <p> <math>\sum M_{C1} = 0</math>  <math>\therefore (A_y \times 3a) - (A_x \times 2h) + (J_y \times a) - (J_x \times 2h) - (W_3 \times 2a) - M_p - M_p = 0 \text{ ----- } \{i\}</math> </p> <p><i>Substituting the values of <math>A_y</math> and <math>A_x</math> (from equations {5} and {4} respectively) into equation {i} above we have;</i>  <math>\therefore aJ_y - 2hJ_x = 7M_p \text{ ----- } \{6\}</math> </p>	



	 <p style="text-align: center;"><b>Fig. 4.27</b></p> $+\sum M_{B1} = 0$ $(G_x \times 2h) - (G_y \times 2a) - (W_3 \times a) = 0$ $\therefore G_y = \frac{W_3 a + M_p}{2a} \text{ (kN)} \text{ ----- \{12\}}$	$G_y = \frac{W_3 a + M_p}{2a} \text{ (kN)}$
<b>Member ref.</b>	<b>Calculation</b>	<b>Output</b>
	<p>Considering the equilibrium of the right-hand side of the frame at point E and D<sub>2</sub>;</p>  <p style="text-align: center;"><b>Fig. 4.28</b></p> $+\sum M_E = 0$ $(H_x \times 2h) + M_p - M_p = 0$ $\therefore H_x = 0 \text{ (kN)} \text{ ----- \{13\}}$	$H_x = 0 \text{ (kN)}$
	 <p style="text-align: center;"><b>Fig. 4.29</b></p> $+\sum M_{D2} = 0$ $\therefore (G_x \times 2h) - (G_y \times 4a) - (H_y \times 2a) + (H_x \times 2h) + (W_3 \times 3a) - (W_3 \times a) - 2M_p - M_p = 0$ <p>Substituting the values of <math>G_x</math>, <math>G_y</math> and <math>H_x</math> in the equation above we have;</p> $\therefore H_y = \frac{2W_3 a - 4M_p}{2a} \text{ (kN)} \text{ ----- \{14\}}$ <p>Considering the equilibrium of forces on the vertical axis to determine <math>I_y</math> and the equilibrium of forces on the horizontal axis to determine <math>I_x</math> we</p>	$H_y = \frac{2W_3 a - 4M_p}{2a} \text{ (kN)}$

	have;	
Member ref.	Calculation	Output
	<p><math>\uparrow \sum F_y = 0</math>  <math>A_y + J_y + I_y + H_y + G_y = 4(W_3)</math>  Substituting the values of <math>A_y, J_y, H_y</math> and <math>G_y</math> as gotten above into the equation we have;  <math>\therefore I_y = \frac{W_3 a + 7M_p}{2a} \text{ (kN)} \text{ ----- } \{15\}</math></p> <p><math>\rightarrow \sum F_x = 0</math>  <math>A_x + J_x + I_x - H_x - G_x + (W_4) = 0</math>  Substituting the values of <math>A_x, J_x, H_x</math> and <math>G_x</math> as gotten above into the equation we have;  <math>\therefore I_x = \frac{11M_p - 2W_3 a - 2W_4 h}{2h} \text{ (kN)} \text{ ----- } \{16\}</math></p> <p>Checking for the bending moment at all points of possible hinges to ensure they are not greater than the <math>M_p</math> value chosen;</p> <p style="text-align: center;"><u><b><math>M_p</math> at point B</b></u></p>  <p style="text-align: center;"><b>Fig. 4.30</b></p> <p><math>\curvearrowright \sum M_B = 0</math>  <math>(-A_x \times 2h) - M_p = 0 \text{ (Substituting the value of } A_x \text{)}</math>  <math>\therefore M_B = 4M_p - W_3 a \text{ (kNm)} \text{ ----- } \{17\}</math></p> <p style="text-align: center;"><u><b><math>M_p</math> at point E<sup>I</sup></b></u></p>  <p style="text-align: center;"><b>Fig. 4.31</b></p>	$I_y = \frac{W_3 a + 7M_p}{2a} \text{ (kN)}$  $I_x = \frac{11M_p - 2W_3 a - 2W_4 h}{2h} \text{ (kN)}$  $M_B = 4M_p - W_3 a \text{ (kNm)}$
	<p><math>\curvearrowright \sum M_{E1} = 0</math>  <math>(G_x \times 2h) - (G_y \times a) = 0 \text{ (Substituting the values of } G_x \text{ and } G_y \text{)}</math>  <math>\therefore M_{E1} = \frac{3M_p - W_3 a}{2} \text{ (kNm)} \text{ ----- } \{18\}</math></p> <p style="text-align: center;"><u><b><math>M_p</math> at point D<sup>I</sup></b></u></p>	$M_{E1} = \frac{3M_p - W_3 a}{2} \text{ (kNm)}$

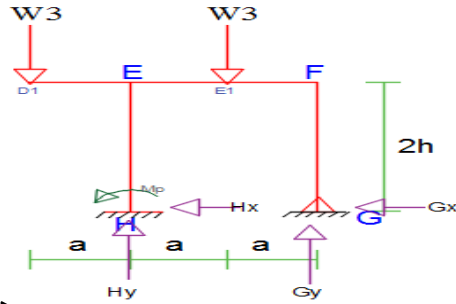


Fig. 4.32

$$\sum M_{D1} = 0$$

$$\therefore (G_x \times 2h) - (G_y \times 3a) - (H_y \times a) + (H_x \times 2h) + (W_3 \times 2a) - M_p = 0$$

Substituting the values of  $G_x$ ,  $G_y$ ,  $H_y$  and  $H_x$  in the equation above we have;

$$\therefore M_{D1} = \frac{M_p - W_3 a}{2} \text{ (kNm)} \text{ ----- \{19\}}$$

Mp at point E1

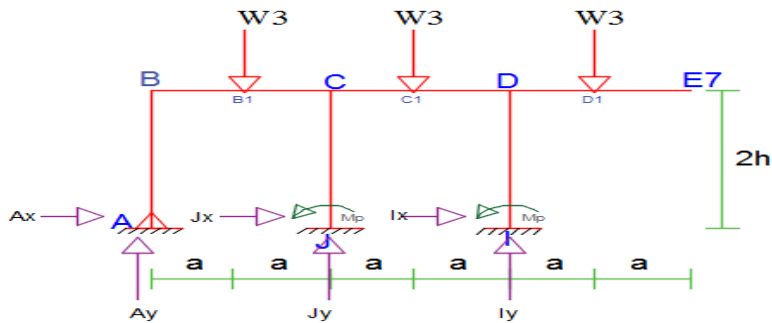


Fig. 4.33

$$\sum M_{E7} = 0$$

$$\begin{aligned} \therefore (A_y \times 6a) - (A_x \times 2h) + (J_y \times 4a) - (J_x \times 2h) + (I_y \times 2a) \\ - (I_x \times 2h) - (W_3 \times 5a) - (W_3 \times 3a) - (W_3 \times a) \\ - 2M_p = 0 \end{aligned}$$

Substituting the values of  $A_x$ ,  $A_y$ ,  $J_x$ ,  $J_y$ ,  $I_y$  and  $I_x$  in the equation above we have;

$$\therefore M_{E1} = 2W_3 a + 2W_4 h - 18M_p \text{ (kNm)} \text{ ----- \{20\}}$$

Mp at point C3

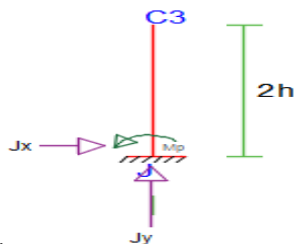


Fig. 4.34

$$\sum M_{C3} = 0$$

$$(-J_x \times 2h) - M_p = 0 \text{ (Substituting the value of } J_x)$$

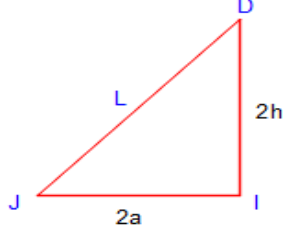
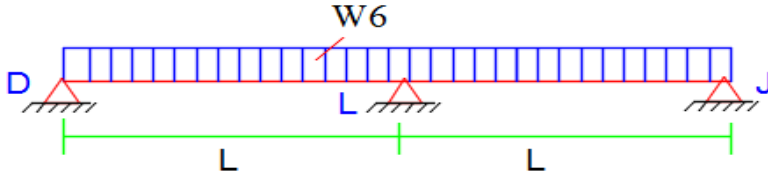
$$\therefore M_{C3} = 5M_p - W_3 a \text{ (kNm)} \text{ ----- \{21\}}$$

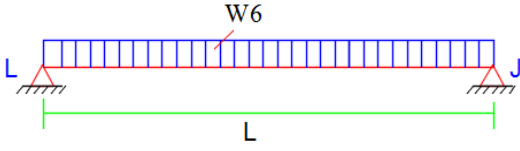
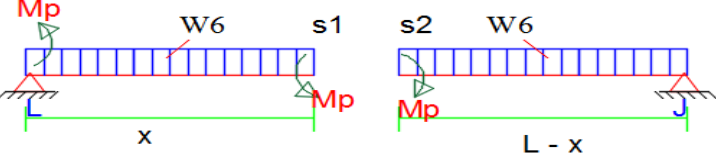
$$\begin{aligned} M_{D1} \\ = \frac{M_p - W_3 a}{2} \text{ (kNm)} \end{aligned}$$

$$\begin{aligned} M_{E7} \\ = 2W_3 a + 2W_4 h \\ - 18M_p \text{ (kNm)} \end{aligned}$$

$$\begin{aligned} M_{C3} &= 5M_p - \\ &W_3 a \text{ (kNm)} \end{aligned}$$



<b>Member ref.</b>	<b>Calculation</b>	<b>Output</b>
	<p><b>4.2.1 DETERMINATION OF THE PLASTIC MOMENT IN MEMBER DLJ (Bracing members)</b></p>  <p style="text-align: center;"><b>Fig. 4.35</b></p> <p>From pythagora's theorem; <math>DJ = \sqrt{(2h)^2 + (2a)^2}</math>  <math>DJ = 2\sqrt{h^2 + a^2}(\text{m})</math> ----- {22}</p> <p style="text-align: center;"><b>Analyzing member DLJ as a beam mechanism</b></p> <p>Let <math>\sqrt{h^2 + a^2}</math> be denoted as "L"          Let the load acting on the bracing members be <math>W_6</math> (kN/m)</p>  <p style="text-align: center;"><b>Fig. 4.36</b></p> <p>No of degree of indeterminacy(<math>I_D</math>) = <math>(2m + r) - 2n</math>          Where; <b>m</b> (number of members) = 2  <b>r</b>(number of reactions) = 3  <b>n</b> (number of nodal points) = 3  <math>(I_D) = ((2 \times 2) + 3) - (2 \times 3) = 1</math>          No of possible position of hinges = 3          @ Joint L, udl between point D and L and udl between point L and J</p>	$DJ = 2\sqrt{h^2 + a^2}(\text{m})$

	<p>No of independent collapse mechanism= <math>3 - 1 = 2</math>                  Beam mechanisms– Beam B – C and                  – Beam C – D</p>	
	<p style="text-align: center;"><b><u>Collapse mechanism 1 – BEAM L – J</u></b></p>  <p style="text-align: right;"><b>Fig. 4.37</b></p> <p>Let <math>\sqrt{h^2 + a^2}</math> be “L”                  Plastic hinge occurs at the position of maximum moment. Cutting a section as shown below to determine the position of the maximum moment.                  Let “x” be the position of maximum moment from point L and  <math>(\sqrt{h^2 + a^2}) - x</math> be L – x</p>  <p><b>Fig. 4.38</b></p> <p><math>\sum M_L = 0</math>  <math>-M_p + \left(W_6 \times \frac{x^2}{2}\right) - M_p = 0</math>  <math>\therefore M_p = \frac{W_6 x^2}{4} \text{ (kNm)} \text{ ----- \{23\}}</math></p> <p><math>\sum M_J = 0</math>  <math>M_p - \frac{W_6}{2}(L - x)^2 = 0</math>  <math>\therefore M_p = \frac{W_6}{2}(L^2 - 2Lx + x^2) \text{ (kNm)} \text{ ----- \{24\}}</math></p> <p style="text-align: center;"><i>Equating equation {23} and {24}</i>  <math>0.25W_6x^2 = 0.5W_6(L^2 - 2Lx + x^2) \text{ (making } W_6 = 1)</math>  <math>\therefore 0.25x^2 - Lx + 0.5L^2</math>                  Applying Almighty formula; <math>x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}</math>, <math>a = 0.25, b = -L, c = 0.5</math></p> $= \frac{-(-L) \pm \sqrt{(-L)^2 - (4 \times 0.25 \times 0.5L^2)}}{2 \times 0.25}$	
	<p style="text-align: center;"><math>\therefore x = 0.586L</math> and  <math>L - x = L - 0.586L = 0.414L</math></p>	

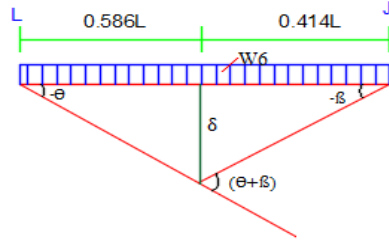


Fig. 4.39

$$\partial = 0.586L\theta = 0.414L\beta$$

$$\therefore \beta = 1.415\theta$$

$$\text{Internal work done (I.W)} = Mp(\theta) + Mp(\theta + \beta)$$

$$\therefore Mp(\theta) + Mp(\theta + 1.415\theta) = 3.415Mp\theta$$

$$\text{External work done (E.W)} = W_6 \times L \times \frac{\partial}{2}$$

$$\therefore W_6 \times L \times \frac{0.586L\theta}{2}$$

$$= \frac{0.586W_6L^2\theta}{2} \text{ (kNm)}$$

Considering work equation: Internal work done = External work done

$$= 3.415Mp\theta = \frac{0.586W_6L^2\theta}{2} \text{ (kNm)}$$

$$\therefore Mp = 0.0932W_6L^2 \text{ (kNm)}$$

Substituting the value of L in the equation above we have;

$$Mp = 0.0932W_6(h^2 + a^2) \text{ (kNm)} \text{----- \{25\}}$$

$$Mp = 0.0932W_6(h^2 + a^2) \text{ (kNm)}$$

Due to symmetry, member D-L = member L-J and as such have equal Mp

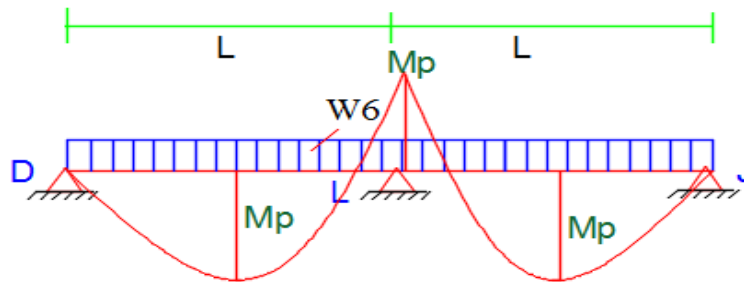


Fig. 4.39b: Bending moment diagram for member DLJ

#### 4.2.2 DETERMINATION OF THE PLASTIC MOMENT IN MEMBER A-G (The bottom cord)

Let the total load acting on the lower cord be  $W_7$  (kN/m)

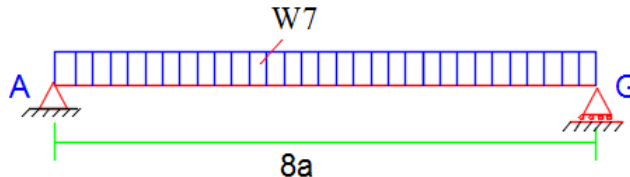


Fig. 4.40

$$\text{No of degree of indeterminacy (I}_D\text{)} = (2m + r) - 2n$$

$$\text{Where; } m \text{ (number of members)} = 1$$

$$r \text{ (number of reactions)} = 2$$

$$n \text{ (number of nodal points)} = 2$$

$$(I_D) = ((2 \times 1) + 2) - (2 \times 2) = 0$$

$$\text{No of possible position of hinges to cause collapse; } I_D + 1 = 0 + 1 = 1$$

@ any point under the udl between point A and G (i.e at the middle due to

udl and no moment at the end reactions)

No of independent collapse mechanism= 1

Beam mechanisms– Beam A – G

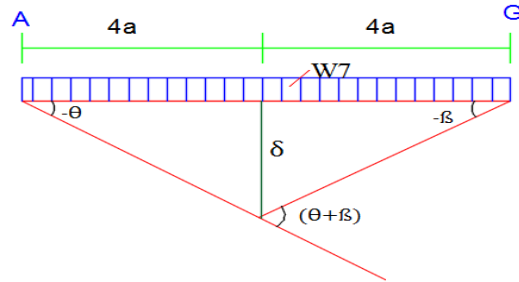


Fig. 4.41

$$\delta = a\theta = a\beta$$

$$\therefore \theta = \beta$$

$$\text{Internal work done (I.W)} = Mp(\theta + \beta) \\ = 2Mp\theta$$

$$\text{External work done (E.W)} = W_7 \times 8a \times \frac{\delta}{2} \\ = W_7 \times 8a \times \frac{4a\theta}{2} \\ = 16W_7a^2\theta \text{ (kNm)}$$

Considering work equation: Internal work done = External work done

$$2Mp\theta = 16W_7a^2\theta \text{ (kNm)}$$

$$\therefore Mp = 8W_7a^2 \text{ (kNm)} \text{----- \{26\}}$$

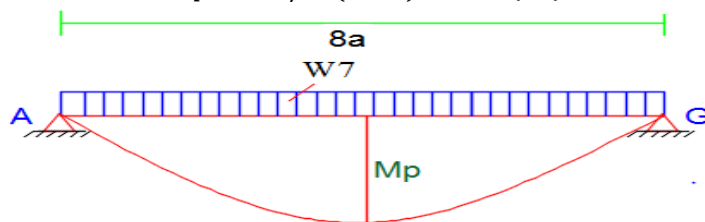
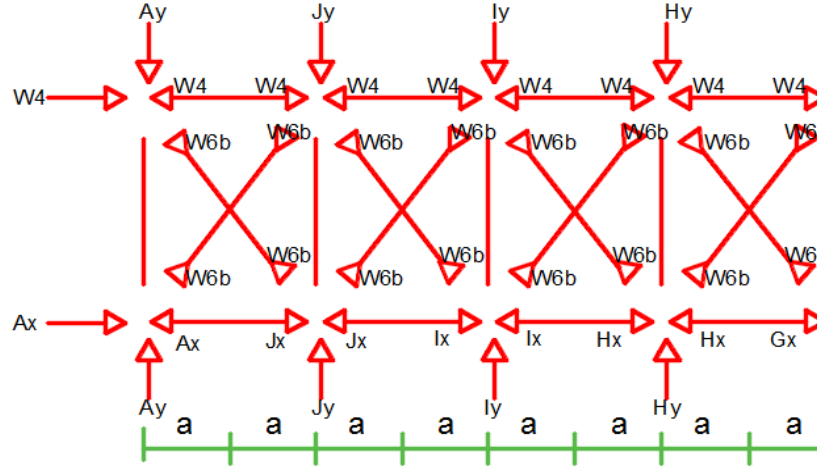


Fig. 4.41b: Bending moment diagram for member A - G

### AXIAL FORCE DIAGRAM

$$\therefore Mp \\ = 8W_7a^2 \text{ (kNm)}$$

	 <p>The diagram shows a truss structure with vertical loads \$A_y, J_y, I_y, H_y\$ and horizontal loads \$W_4\$ acting on the top chord. Reactions \$A_x, J_x, I_x, H_x, G_x\$ are shown at the bottom. Internal bracing members are labeled \$W_6b\$. The horizontal distance between vertical load points is \$a\$.</p> <p>the axial forces acting on the members of the complex truss system</p> <p><b>N.B:</b> The horizontal and vertical forces acting on the internal bracing members were resolved in the axis on the bracing members to determine their axial forces.</p>	
<p>5.1.1 EC3.</p>	<p><b>4.3.1 DESIGN METHOD FOR TOP CHORD AND BOTTOM CHORD (BEAM SECTION)</b></p> <p>Beam span = L (8a)m          Bay width = 2a</p> <p><b>Actions</b>          Dead load = <math>G_k</math> =          Live load (<math>Q_k</math>) = <math>W_7</math> kN          Design value of combined actions =  <math>W_8</math> kN/m<sup>2</sup>          UDL per meter length of beam accounting for bay width of 2a = <math>W_8 \times 2a</math>  <math>= 2aW_8 = W_9</math> kN/m</p> <p><b>Design Moment And Shear Force</b>          Maximum design moment <math>M_{y,Ed}</math> occurs at mid-span and for bending about the major (y-y)-axis is <math>M_{y,Ed} = \frac{W_9 \times L^2}{8} = \frac{W_9 \times (8a)^2}{8} = 8W_9a^2</math> kNm ----- {B1}          Maximum design shear force <math>V_{Ed}</math> occurs at the end supports, and is;  <math>V_{Ed} = \frac{W_9 \times L}{2} = \frac{W_9 \times 8a}{2} = 4W_9a</math> kN ----- {B2}</p> <p><b>Partial factors for resistance</b>  <math>\gamma_{M0} = 1.0</math></p> <p><b>Trial Section</b>          Yield strength = <math>f_y</math> N/mm<sup>2</sup>          (lets choose <math>f_y</math> to be 275 N/mm<sup>2</sup> if <math>t_w &lt; 16</math> mm and for the purpose of explanation in this project)          The required section needs to have plastic modules about the major-axis (y-y) that is greater than:  <math>S_y = \frac{M_{y,Ed} \times \gamma_{M0}}{f_y} = \frac{8W_9a^2 \times 10^3 \times 1.0}{275}</math> (cm<sup>3</sup>) ----- {B3}</p>	<p>UDL = <math>W_9</math> (kN)</p> <p><math>M_{y,Ed} = 8W_9a^2</math> kNm</p> <p><math>V_{Ed} = 4W_9a</math> kN</p>

		$S_x = \frac{8W_9 a^2 \times 10^3 \times 1.0}{275}$ (cm <sup>3</sup> )
3.2.6 (1)  Table 5.3.1 of EC3	<p>Choose a section from table ... with plastic modulus &gt;S<sub>y</sub> and select its properties.</p> <p>Depth of cross-section = D mm</p> <p>Web depth = h<sub>w</sub> mm (<math>h_w = h - 2t_f</math>)</p> <p>Width of cross-section = B mm</p> <p>Depth between fillets = d mm</p> <p>Web thickness = t<sub>w</sub> mm</p> <p>Flange thickness = t<sub>f</sub> mm</p> <p>Radius of root fillet = r mm</p> <p>Cross-sectional area = A cm<sup>2</sup></p> <p>Second moment of area (y-y) = I<sub>y</sub> cm<sup>4</sup></p> <p>Second moment of area (x-x) = I<sub>x</sub> cm<sup>4</sup></p> <p>Elastic section modulus (y-y)= Z<sub>ey</sub>cm<sup>3</sup></p> <p>Plastic section modulus (y-y)=Z<sub>py</sub>cm<sup>3</sup></p> <p>Take modulus of elasticity E to be 210000 N/mm<sup>2</sup>(for the purpose of explanation)</p> <p><b><u>Classification of cross-section</u></b></p> <p><math>\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.92</math></p> <p><b><u>Outstand flange under uniform compression</u></b></p> <p><math>c = \frac{(B-t_w-2r)}{2} \therefore \frac{c}{t_f} = k_f</math> ----- {B4}</p> <p>From table 5.3 of EC3, check which class the flange section falls in.</p> <p><b><u>Internal compression part: (web under pure bending)</u></b></p> <p><math>c = d \therefore k_w = \frac{d}{t_w}</math> ----- {B5}</p> <p>Also check for the class of the section in the table.</p>	$\frac{c}{t_f} = k_f$ $k_w = \frac{d}{t_w}$
6.2.6 (6)	<p><b><u>N.B:</u></b>Both of the flange and the web must fall in one class of section, less another section that satisfies the condition will be chosen.</p> <p>We can also go directly to the table (1) and choose the ratios for the local buckling for flange and web grade them using table 3.</p> <p><b><u>Shear buckling</u></b></p> <p>Shear buckling of the unstiffened web need not be considered provided:</p> <p><math>\frac{h_w}{t_w} \leq 72 \frac{e}{n} \therefore n = 1.0</math> (conservative)</p> <p><b><u>Shear capacity</u></b></p> <p><math>W_9 \leq P_v \quad \therefore \frac{W_9}{P_v} \leq 1.0</math></p>	

[illegible]



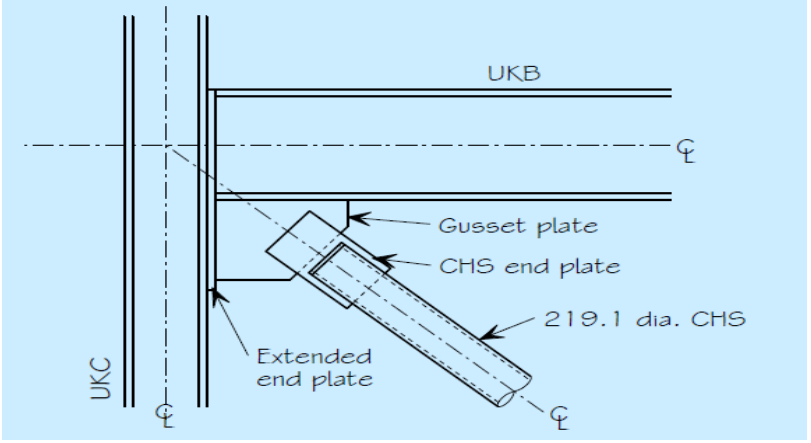
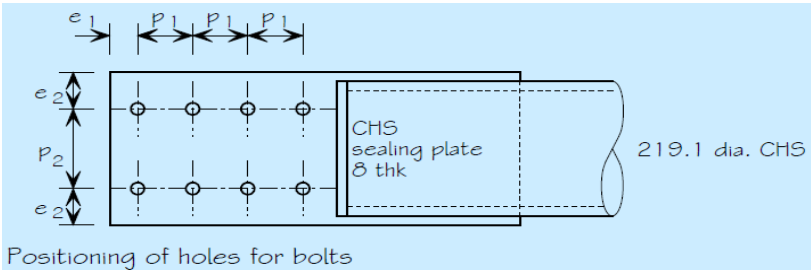
6.1(1) NA 2.15	<p><b>4.3.2 DESIGN OF VERTICAL MEMBERS (COLUMNS/STRUTS)</b> <b>(COMPRESSION MEMBERS)</b></p> <p>Moment = M  <math>S_y = \frac{M}{f_y} (\text{cm}^3) \text{ where } f_y = 275 \text{ N/mm}^2</math></p> <p><u>Data</u>  Axial force = N  Design moment (<math>M_i</math>)= <math>\frac{M}{2}</math> ----- { C1 }</p> <p><u>Partial factors for resistance</u>  <math>Y_{M0} = 1.05</math>  <math>Y_{M1} = 1.0</math></p> <p><u>Trial section</u>  Initial trial section is selected to give a suitable moment capacity. The size is then checked to ensure suitability in all other aspects.  <i>Choose a section from table... with plastic modulus &gt; <math>S_y</math> and select its properties.</i></p> <p><u>Section Properties</u>  Depth of cross-section = D mm  Width of cross-section = B mm  Depth between fillets = d mm  Web thickness = t_w mm  Flange thickness = t_f mm  Cross-sectional area = A cm<sup>2</sup>  Second moment of area (y-y) = I_y cm<sup>4</sup>  Elastic section modulus (y-y) = Z_ey cm<sup>3</sup>  Plastic section modulus (y-y)=Z_py cm<sup>3</sup></p>	<p><math>S_y = \frac{M}{f_y} (\text{cm}^3)</math></p> <p><math>M_i = \frac{M}{2}</math></p>
SN048a-	<p><u>In-plane failure about major axis</u>  Members subject to axial compression and major axis bending must</p>	





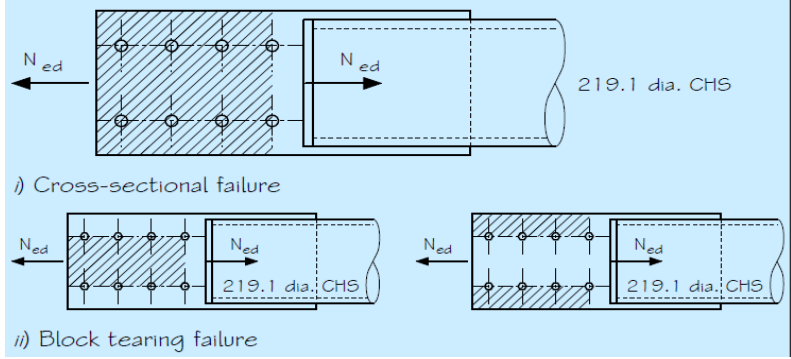
<p>NA 2.4 BS EN 10210-1 Table A3</p> <p>Table 5.3.1</p> <p>6.2.4(1) Eq. 6.9</p> <p>6.2.4(2) Eq. 6.10</p> <p>6.3.1.1(1) Eq. 6.46</p>	<p><b>Material properties</b> Steel grade = S355 If <math>t \leq 16</math> mm, then, Yield strength <math>f_y = 355</math> N/mm<sup>2</sup> 3.2.6 (1) modulus of elasticity <math>E = 210</math> kN/mm<sup>2</sup></p> <p><b>Section classification</b></p> $\varepsilon = \sqrt{\frac{235}{f_y}} \text{-----} \{D4\}$ <p>Check for the classification of section in EC3 table 5.3.1</p> <p><b>Design of member in compression</b> <b>Cross sectional resistance to axial compression</b> Basic requirement; <math>\frac{N}{N_{c,Rd}} \leq 1.0</math>----- {D5}</p> <p><math>N</math> - is the design value of the applied axial force <math>N_{c,Rd}</math> - is the design resistance of the cross-section for uniform compression.</p> <p>where;</p> $N_{c,Rd} = \frac{A \times f_y}{\gamma_{M0}} \text{(For Class 1, 2 and 3 cross-sections) -----} \{D6\}$ <p>If equation D5 is satisfied, then the resistance of the cross-section is adequate.</p> <p><b>Flexural buckling resistance</b> For a uniform member under axial compression the basic requirement is:</p> $\frac{N}{N_{b,Rd}} \leq 1.0 \text{-----} \{D7\}$ <p><math>N_{b,Rd}</math> - is the design buckling resistance and is determined from;</p> $N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \text{(For Class 1, 2 and 3 cross-sections) -----} \{D8\}$	

<p>6.3.1.2(1) Table 6.2</p> <p>Table 5.2</p> <p>Table 5.5.2</p> <p>6.2.3</p>	<p><math>\chi</math> is the reduction factor for buckling and may be determined from Figure 6.4. For hot finished CHS in grade S355 steel use buckling curve 'a' For flexural buckling the slenderness is determined from:</p> $\lambda = \sqrt{\frac{Af_y}{N_{cr}}} = \left(\frac{L_{cr}}{i}\right) \left(\frac{1}{\lambda_1}\right) \text{ (For Class 1, 2 and 3 cross-sections)}$ <p>As the bracing member is pinned at both ends, conservatively take:  <math>L_{cr} = L = \sqrt{(2a)^2 + (2h)^2}</math></p> <p>where;  <math>L_{cr}</math> = is the buckling length  <math>i</math> = is the radius of gyration</p> <p><math>\lambda_1 = 93.9\epsilon</math>  <math>\therefore \lambda = \left[ \frac{\sqrt{(2a)^2 + (2h)^2}}{i} \right] \left[ \frac{1}{93.9\epsilon} \right] \text{----- \{D9\}}</math></p> <p>From buckling curve 'a', find the equivalent value of <math>\chi</math> that corresponds with the value of <math>\lambda</math> gotten in equation D8. If equation D7 is satisfied, then the flexural buckling resistance of the section is adequate.</p> <p><b>Design of member in tension</b> When the wind is applied in the opposite direction, the bracing member considered above will be loaded in tension. By inspection, the tensile capacity is equal to the cross-sectional resistance.</p>	<p>Use buckling curve 'a'</p>
	<p align="center"><b><u>RESISTANCE OF CONNECTION</u></b></p> <p>Assume the CHS is connected to the frame via gusset plates. Flat end plates fit into slots in the CHS section and are fillet welded to the CHS. Bolts in clearance holes transfer the load between the end plate and gusset plates. Verify the connection resistance "N" kN tensile force.</p>	

<p>P363</p> <p>BS EN 1993-1-8 Table 3.1</p>	 <p><b>Fig. 4.43: Bracing setting out and connection detail</b></p> <p>Try: 8 No non-preloaded Class 8.8 M24 diameter bolts in 26 mmdiameter clearance holes.</p> <p>Assume shear plane passes through the threaded part of the bolt</p> <p>Cross section area, <math>= A \text{ mm}^2</math></p> <p>Clearance hole diameter, <math>d_0 = 26 \text{ mm}</math></p> <p><i>For Class 8.8 non-preloaded bolts:</i></p> <p>Yield strength <math>f_{yb} = 640 \text{ N/mm}^2</math></p> <p>Ultimate tensile strength <math>f_{ub} = 800 \text{ N/mm}^2</math></p>	
<p>BS EN 1993-1-8 Figure 3.1</p> <p>BS EN 1993-1-8</p>	<p><b>Positioning of holes for bolts:</b></p> <p>(Minimum) End distance (<math>e_1</math>) <math>= 1.2 d_0 &lt; e_1 = 40 \text{ mm}</math></p> <p>(Minimum) Edge distance (<math>e_2</math>) <math>= 1.2 d_0 &lt; e_2 = 60 \text{ mm}</math></p> <p>(Minimum) Spacing (<math>p_1</math>) <math>= 2.2 d_0 &lt; p_1 = 80 \text{ mm}</math></p> <p>(Minimum) Spacing (<math>p_2</math>) <math>= 2.4 d_0 &lt; p_2 = 130 \text{ mm}</math></p> <p>(Maximum) <math>e_1</math> and <math>e_2</math>, larger of <math>8t &gt; 40 \text{ mm}</math> and <math>60 \text{ mm}</math></p> <p>(Maximum) <math>p_1</math> and <math>p_2</math> Smaller of <math>14t &gt; 80 \text{ mm}</math> and <math>130 \text{ mm}</math></p> <p><i>If satisfied, then bolt spacings comply with the limits.</i></p>  <p><b>Fig. 4.44: Positioning of holes for bolts</b></p> <p>Choose the grade and section of end plate thick to fit into a slotted hole in the CHS</p> <p><b>Shear resistance of bolts:</b></p> <p>The resistance of a single bolt in shear is determined from:</p> $F_{v,Rd} = \frac{a_v f_{ub} A}{\gamma_{M2}} \text{ ----- } \{D10\}$ <p>Where; <math>a_v = 0.6</math> for grade 8.8 bolts</p> <p>Minimum number of bolts “n” required is;</p>	

**COMPLEX TRUSS ANALOGY USING PLASTIC AND ELASTIC ANALYSIS**

Table 3.4	$\frac{N}{F_{v,Rd}} = 'n'$ bolts ----- {D11} <i>Then provide no of bolts &gt; n number of bolts gotten in equation D11 above.</i>	$F_{v,Rd} = \frac{a_v f_{ub} A}{\gamma_{M2}}$ $\frac{N}{F_{v,Rd}} = n$
BS EN 1993-1-1 NA 2.4 BS EN 10025-2 Table 7  BS EN 1993-1-8 Table 3.4	<p><b><u>Bearing resistance of bolts</u></b></p> <p>Assume gusset plate has a thickness no less than the 15 mm endplate.            End plate is a grade S275 and if <math>t \leq 16</math> mm, for S275 steel, then yield strength, <math>f_y = 275</math> N/mm<sup>2</sup>            if <math>3 \leq t \leq 100</math> mm; then ultimate tensile strength <math>f_u = 410</math> N/mm<sup>2</sup></p> <p>The bearing resistance of a single bolt is determined from;  <math display="block">F_{b,Rd} = \frac{k_1 a_b f_u d t}{\gamma_{M2}} \text{ ----- {D12}}</math> <i>Where;</i>  <math>a_b</math> is the least value of <math>\alpha_d, \frac{f_{ub}}{f_{u,p}}</math>, and 1.0            For end bolts, <math>\alpha_d = \frac{e_1}{3d_0}</math> ----- {D13}            For inner bolts, <math>\alpha_d = \frac{e_1}{3d_0} - \frac{1}{4}</math> ----- {D14}  <math>\frac{f_{ub}}{f_{u,p}} = \alpha_z</math> ----- {D15}  <i>The lowest among the values gotten from equations D13, D14, D15 and 1.0 is taken as the value of “<math>a_b</math>”</i></p> <p>For edge bolts <math>k_1</math> is the smaller of; <math>2.8 \frac{e_2}{d_0} - 1.7</math> or 2.5            For inner bolts <math>k_1</math> is the smaller of; <math>1.4 \frac{p_2}{d_0} - 1.7</math> or 2.5  <i>Therefore, choose the value of <math>k_1</math> from above and substitute the values gotten in equation D12</i>            Resistance of all six bolts in bearing may be conservatively taken as:  <math>8 \times F_{b,Rd}</math> kN</p>	$F_{b,Rd} = \frac{k_1 a_b f_u d t}{\gamma_{M2}}$
BS EN 1993-1-8 3.7	<p><b><u>Group of fasteners</u></b></p> <p>Because the shear resistance of the bolts is less than the minimum bearing resistance of any bolt, the design resistance of the group is taken as:  <math>8 \times F_{v,Rd} = F_{Gp}</math> (kN)</p>	Resistance of the bolt group = $F_{Gp}$ (kN)

<p>BS EN 1993-1-8 3.10.2</p> <p>6.2.3(1)</p> <p>6.2.3(2)</p> <p>Eqn. 6.6</p>	<p><b><u>Tensile resistance of end plate (see Figure 9.4)</u></b> Two modes of failure are to be considered: i) Cross-sectional failure and ii) Block tearing failure.</p>  <p><b>Fig. 4.45: Plate failure modes</b></p> <p><b>i) Cross-sectional failure:</b> Basic requirement: <math>\frac{N}{N_{t,Rd}} \leq 1.0</math> ----- {D16} For a cross-section with holes, the design tensile resistance is taken as the smaller of <math>N_{pl,Rd}</math> and <math>N_{u,Rd}</math>: <math>N_{pl,Rd} = \frac{A_g \times f_y}{\gamma_{M0}}</math> ----- {D17} Where; <math>A_g = A \times t_p</math> (the gross cross-sectional area) If <math>N_{pl,Rd} &gt; N</math>, then OK</p>	$N_{pl,Rd} = \frac{A_g f_y}{\gamma_{M0}}$
<p>Eqn. 6.7</p> <p>6.2.2.2</p> <p>BS EN 1993-1-8 3.10.2 (2)</p>	<p><math>N_{u,Rd} = \frac{0.9 \times A_{net} \times f_u}{\gamma_{M2}}</math> ----- {D18} Where; <math>A_{net} = A - (2 \times d_0 \times t_p)</math> here <math>t_p = 15</math> If <math>N_{u,Rd} &gt; N</math> then, OK</p> <p><b>ii) Block tearing failure</b> For a symmetric bolt group subject to concentric loading, the design block tearing resistance (<math>V_{Eff,1,Rd}</math>) is determined from: <math>V_{Eff,1,Rd} = \frac{f_u A_{nt}}{\gamma_{M2}} + \left(\frac{1}{\sqrt{3}}\right) \left(\frac{f_y A_{nv}}{\gamma_{M0}}\right)</math> ----- {D19} Where; <math>A_{nt}</math> is the net area subject to tension <math>A_{nv}</math> is the net area subject to shear <math>A_{nt}</math> is the minimum of <math>(P_2 - d_0)t_p</math> and <math>2(e_2 - 0.5d_0)t_p</math> and <math>A_{nv} = 2((3p_1 + e_1) - (2.5d_0))t_p</math> Substitute the values of <math>A_{nt}</math> and <math>A_{nv}</math> in equation D16 If <math>V_{Eff,1,Rd} &gt; N</math> then, OK</p>	$\begin{aligned} \frac{N_{u,Rd}}{0.9 A_{net} f_u} &= \frac{1}{\gamma_{M2}} \\ V_{Eff,1,Rd} &= \frac{f_u A_{nt}}{\gamma_{M2}} + \left(\frac{1}{\sqrt{3}}\right) \left(\frac{f_y A_{nv}}{\gamma_{M0}}\right) \end{aligned}$



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### **Discussion and Conclusion:**

the summary for the study on the comparison between elastic analysis and plastic analysis for the design of complex truss.

In this lesson we have studied how the loads are transferred in bridge truss floor system. Further, we found that there is similarity between the influence line of support reactions for simply supported beam and truss structures. Finally we studied the influence line for truss member forces.

Influence lines as we have seen is a function whose value at any given point represents the value of some structural quantity due to a unit force placed at that point. The influence line graphically shows how changing the position of a single load influences various significant structural quantities. (Structural quantities: Reactions, Shear, Moment, Deflection, etc.)

Influence lines may be used to advantage in the determination of simple beam reactions. In this case, the use of the unit influence line is necessary. The unit influence line represents the effects of unit: reactions (displacements), shears (separations) and moments (rotations) in a beam structure.

We have also seen how plastic method of analysis can be used to analyse not just truss systems but the complex truss systems. Unpinning the members of the truss system makes it very much easier and very much explanatory in the analysis of the truss systems.

Plastic analysis of the complex truss system as we have seen agrees to the theory of plasticity which says that a structure is deemed to have reached the limits of its load bearing capacity when it forms sufficient hinges to convert it to a mechanism with consequent collapse. This is normally one hinge more than number of degree of indeterminacy ( $I_p$ ).

The plastic collapse loads corresponding to various failure mechanisms as we have seen are obtained by equating the internal work at the plastic hinges to the external by loads during the virtual displacement. This requires evaluation of displacements and plastic hinge rotations.

During the last few decades, computer software has become more and more critical in the analysis of engineering and scientific problems. Much of the reason for this change from manual methods has been the advancement of computer techniques developed by the research community and, in particular, universities.

As both the Technology and Engineering industries advance, new methodologies of interlinking and complementing the industries via computer applications will be created, with a similar improvement in hardware capacities. This in turn will facilitate the implementation of more efficient and professional engineering software. As these software applications advance in functionality, one can hope that they will be more affordable so as to promote their widespread usage amongst civil engineers at a global scale.

The introduction of software usage in the civil engineering industry as we have seen has greatly reduced the complexities of different aspects in the analysis and design of projects, as well as reducing the amount of time necessary to complete the designs. Concurrently, this leads to greater savings and reductions in costs. More complex projects that were almost impossible to work out several years ago are now easily solved with the use of computers. In order to stay at the pinnacle of any industry, one needs to keep at par with the latest technological advancements which accelerate work timeframes and accuracy without decreasing the reliability and efficiency of the results.

### Plastic analysis vs elastic analysis

It may not be realized, but the advantages of plasticity of metal are consciously or unconsciously made use of even in elastic design methods. For example, in the elastic method of design, if a design is too conservative for a given permissible working stress, then the stress value is changed, indicating that plasticity is made use of.

### Advantages of the plastic method of analysis

Normally, there are two distinct advantages of plastic methods over the conventional or elastic methods. Firstly, they are more economical as they make full use of the materials strength beyond the elastic limit. Secondly, the design procedures are much simpler and rational.

It has been observed earlier that metals, especially steel have considerable reserve of strength beyond that elastic limit. Also, ultimate load for these can be computed more precisely and accurately. Taking advantage of the above, the plastic methods permit use of much smaller structural section to safely support the working loads. As regards simplicity of procedures, the plastic methods are inherently simple, as they do not take consideration the elastic conditions of continuity, which involves tedious and complicated calculations. It is for these reasons that plastic design methods are calculated.

In the influence line analysis, the mobile load acting on the truss system was directly applied in the determination of the axial forces acting on the respective truss members where as for the plastic analysis method the mobile load has to be multiplied with the section's load factor before analysis.

From the influence line analysis, I observed that the top chord members of the truss system are all compression members, the lower chord members are all tension members then both the vertical strut and braced members are being acted upon by both compression and tension forces.

The internal braced members of the truss system exist both in primary and secondary truss and has to be designed accordingly. I also noticed that after analysis, the result showed that the braced members have the same magnitude of compressive and tension forces.

In the influence line analysis of the complex truss designed in chapter four, the effect of the mobile load on the truss members is highest when the position of the mobile load is at the middle of the truss system. To obtain the maximum value of a function due to a single concentrated live load, the load should be placed at that point where the ordinate to the influence line for that function is a maximum.

The value of a function due to the action of a single concentrated live load equals the product of the magnitude of the load and the ordinate to the influence line for that function, measured at the point of application of load.

It is only when the reduced frame structure is pinned at the both ends and fixed at the internal support that it will satisfy the required number of independent collapse mechanism. From the table of the combined mechanism, It was observed that the highest  $M_p$  value required to induce collapse is  $\frac{2(a^2W_2)+2hW_4}{13} \text{ kNm}$  and occurs at column 13 of the table.

From the results of the reactions obtained in chapter four from the plastic method of truss analysis, the maximum compressive axial force acting on the vertical members occurs at point J ( $J_y$ ) and the maximum tension axial force acting at the lower chord member occurs at point I ( $I_x$ )

When checking for the bending moment at all points of possible hinges, I observed that the  $M_p$ s gotten are higher than the required  $M_p$  gotten from the combined mechanism. Nevertheless, the maximum bending moment which occurs at point  $E_7$  and  $C_3$  will be used as the design moment gotten from the plastic method of analysis. The lower chord and the bracing members have their own respective  $M_p$ s which are  $8W_7a^2 \text{ kNm}$  and  $0.0932W_6(h^2 + a^2) \text{ kNm}$  respectively.

A close examination on the chapter four of this project disputes the advantages listed above in 5.3, when it comes to the analysis of complicated truss systems acted upon by mobile loads. Unlike beams and frames truss systems involve a combination of many members and as such it requires a lot of rigorous processes and assumptions especially when using the plastic analysis method.

On the basis of economy, plastic method of analysis is mainly economical when it comes to the analysis of frames and beams. When it comes to complicated truss systems acted upon by mobile load, the use of influence line is much safer.

Plastic method of analysis does not give a clear effect of the mobile load on each member with respect to the position of the mobile load. The use of influence line analysis gives directly the axial force exerted by the mobile load on each truss member with respect to it's position.

The top chord and the strut (column) members have the same moment (*i.e the maximum  $M_p$  value*) when using plastic method of analysis. With elastic method of analysis the top chord and the strut members do not have the same design moment.

The results obtained from the research of this work shows that the influence line analysis generates higher axial forces on members than with the plastic analysis method under the same magnitude of imposed live

loads. This is so because the plastic method of analysis involves a lot of assumptions that makes it yet not advisable to be used in the analyses of trusses carrying mobile loads.

Influence line for mobile load analysis is easily written in a programmable form because it is easier and gives the required axial forces directly than the plastic method of mobile analysis.

A user-friendly program for the computer analysis of influence line and plastic method of analyzing complicated truss system and design of steel trusses has been successfully created and tested for the following: Trussanalysis with the following variable input parameters:

Span length

Span height

Type and intensity of loading

The program instantaneously calculates and displays the following results using the above parameters:

The total axial forces acting on each member of the truss system for influence line analysis

The maximum  $M_p$  values that will be acting on the truss members for plastic method of analysis

The axial and the shear forces acting on each member when using the plastic method of analysis.

The wind loading at each beam is transferred to two vertically braced end bays on grid lines 'A' and 'J' by the beams acting as diaphragms. The bracing system carries the equivalent horizontal forces (EHF) in addition to the wind loads. Locally, the bracing must carry additional loads due to imperfections at splices (cl 5.3.3(4)) and restraint forces (cl 5.3.2(5)). These imperfections are considered in turn in conjunction with external lateral loads but not at the same time as the EHF. The braced bays, acting as vertical pin-jointed frames, transfer the horizontal wind load to the lower chord members. The beams and columns that make up the bracing system have already been designed for gravity loads<sup>1</sup>). Therefore, only the diagonal members have to be designed and only the forces in these members have to be calculated. All the diagonal members are of the same section, thus, only the most heavily loaded member has to be designed.

Finally, there is always an assumption that trusses cannot be analysed using plastic method of analysis since they (trusses) are subjected to axial forces and not bending. But from the research shown above in chapter four, we have seen that trusses can be analysed using plastic moment analysis if the necessary steps are being followed.

In this work, we have also seen how trusses can be designed using the current code for design regulation, the Euro code 3.

### **Recommendation:**

The recommendations directly affiliated with this project are given as follows:

The use of influence line analysis should be used for the analysis of complicated truss systems carrying mobile loads (e.g. bridge trusses), since it gives directly the design axial forces on each member.

More research or experiment should be made on plastic method of analysis of truss systems acted upon by mobile or static loads in order to discover more benefits of using the plastic method of analysis for truss systems.

Conscientious effort should be made to expose undergraduate students to the use of plastic method of analysis in order to sensitize its use in the would be engineers.

To continue developing, expanding and improving this software application hoping that one day, it will be a full structural analysis program catering for the analysis and design of frames, trusses and other structural elements.

The Department of Civil Engineering at NnamdiAzikiweUniversity should introduce a computer lab for use by students so as to promote the use of computers in the engineering profession.

The department should encourage conducting similar final year projects dealing with computer applications in the future.

More emphasis regarding computer technology and applications to engineering should be made at an academic level in different courses. This would broaden the intellect of students as well as expose them to new technologies in all engineering disciplines.

Civil engineering students should be thought on the use of Euro codes which is the new code for the design of civil engineering structures as against the BS codes.

Modern buildings are being built using steel materials. Students/engineers should be encouraged to learn the design of steel structures (e.g. trusses, frames, buildings e.t.c.) according to EC3 in order to suite the contemporary world.

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