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Bayesian inference for data with Gaussian behavior with unknown mean and variance and its relationship with the Student 's t- test , application to annual maximum concentrations of ozone in Mexico City

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Abstract: A recap will be made on Bayesian inference models with unknown mean (mu) and variance (σ^2). These models use conjugate prior distributions, commonly a Normal-Gamma or Inverse Normal-Gamma distribution, to jointly estimate the parameters and the model when the variance is unknown. A modification is made to find the same parameters jointly. This can be seen in the article "Fundamentals for Obtaining New Probability Distribution Functions with Bayesian Inference with Gaussian and Near Gaussian Behavior," which demonstrates the use of Bayesian inference with only the known variance and modifies the algorithm's input data to estimate the mean. As will be seen below, both methods yield the same result.
Keywords: Bayesian Inference, Normal pdf, t Student pdf, Likelihood, Inverse Normal-Gamma pdf

Introduction

Bayesian inference models with unknown mean (mu) and variance (σ^2σ²) use conjugate prior distributions, commonly a Normal-Gamma or Inverse Normal-Gamma distribution, to jointly estimate the parameters. The posterior distribution is obtained by updating the initial information with the likelihood of the data, allowing the calculation of probability distributions for the unknown parameters. We will now relate this technique to the method previously published in | Volume 07 - Issue 09 | September 2021 | pp. 15-30 http://www.ijlret.com/Papers/Vol-07-issue-09/3.B2021187.pdf Fundamentals for Obtaining New Probability Distribution Functions with Bayesian Inference with Gaussian and Near Gaussian Behavior , where the use of Bayesian inference with only the known variance is shown and the input data of the algorithm is modified to be able to estimate the mean, both methods as will be seen below give the same result.

Bayesian Inference

Bayesian inference is the process of analyzing statistical models by incorporating prior knowledge about the model or its parameters. The foundation of such inference is Bayes ' theorem .

$$P(Parametros|Datos) = \frac{P(Datos|Parametros) * P(Parametros)}{P(Datos)} \approx \approx FVerosimilitud * PDF\ Priori$$

In this case, we have observations in the form of a normal distribution; the distribution is assumed to be known, σbut we do not know the mean, and we want to make inferences about that mean.θ

$$X_{n1}, X_{n2} \dots X_n \dots X|\theta \sim N(\theta, \sigma^2)$$

Where the sigma or its variance is known beforehand and the PDF a priori is

$$\theta \sim N(\mu, \tau^2)$$

Here, mu and tao are also known. We are looking for n samples of the observed data. In the case of Ozone, the maximum values are those above 150 ppb . In the case of PM10 , it is above 150 microgr /m3. In the case of PM2.5, it is above 65 microgr /m3. And in the case of Maximum Temperatures, it is the entire data sample. We obtain a new average mean that tells us the behavior of the temperature. Thus, we obtain the new normal distribution function with the new parameter sought.

Now we proceed to find that new probability distribution function

$$f(\theta|x_1,x_2,x_3,\dots,x_n)=\frac{f(x_1,x_2,x_3,\dots,x_n;\theta)}{f(x_1,x_2,x_3,\dots,x_n)}=\frac{f(x_1,x_2,x_3,\dots,x_n;\theta)f(\theta)}{f(x_1,x_2,x_3,\dots,x_n)}\propto$$

$$FVerosimilitud * PDF\ Priori \ \propto$$

$$f(x_1,x_2,x_3,\dots,x_n;\theta)f(\theta)\ \propto\ \left(\prod_{i=1}^{n} e^{\left\{-\frac{1}{2}\left(\frac{x_i-\theta}{\sigma}\right)^2\right\}}\right)e^{\left\{-\frac{1}{2}\left(\frac{\theta-\mu}{\tau}\right)^2\right\}}\ \propto$$

$$\propto\ \left(-\frac{1}{2}\sum_{i=1}^{n} e^{\left\{-\frac{1}{2}\left(\frac{x_i-\theta}{\sigma}\right)^2\right\}}\right)\left(e^{\left\{-\frac{1}{2}\left(\frac{\theta-\mu}{\tau}\right)^2\right\}}\right)\propto$$

$$\propto\ \left(\exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}\frac{1}{\sigma^2}(x_i^2-2x_i\theta+\theta^2)+\frac{1}{\tau^2}(x_i^2-2x_i\mu+\mu^2)\right]\right\}\right)$$

$$\propto\ \left(\exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{n}\frac{1}{\sigma^2}(-2x_i\theta+\theta^2)+\frac{1}{\tau^2}(\theta^2-2\theta\mu)\right]\right\}\right)$$

$$\propto\ \left(\exp\left\{-\frac{1}{2}\left[\theta^2\left(\frac{n}{\sigma^2}+\frac{1}{\tau^2}\right)-2\theta\left(\frac{\sum_{i=1}^{n}x_i}{\sigma^2}+\frac{\mu}{\tau^2}\right)\right]\right\}\right)$$

$$\propto\ \left(\exp\left\{-\frac{1}{2}\left(\left(\frac{n}{\sigma^2}+\frac{1}{\tau^2}\right)\left(\theta^2-2\theta\left(\frac{\frac{n\bar{x}}{\sigma^2}+\frac{n\mu}{\tau^2}}{\frac{n}{\sigma^2}+\frac{1}{\tau^2}}\right)\right)\right)\right\}\right)$$

This is the new Gaussian Bayesian distribution function

$$\theta|X\ \sim\ N(a,b^2)$$

$$\theta|X\sim NB\left(\frac{\tau^2}{\frac{\sigma^2}{n}+\tau^2}*\bar{x}+\frac{\frac{\sigma^2}{n}}{\frac{\sigma^2}{n}+\tau^2}*\mu,\frac{\frac{\sigma^2}{n}*\tau^2}{\frac{\sigma^2}{n}+\tau^2}\right)$$

With

$$a=\frac{\tau^2}{\frac{\sigma^2}{n}+\tau^2}*\bar{x}+\frac{\frac{\sigma^2}{n}}{\frac{\sigma^2}{n}+\tau^2}\mu b^2=\frac{\frac{\sigma^2}{n}*\tau^2}{\frac{\sigma^2}{n}+\tau^2}$$

In the case of Ozone maximums, the number of data sought is n with a σ^2sample variance, or sample size, where $\bar{x}$is the Gaussian pdf previously adjusted with a mean. $\mu\ y\ una\ desviacion\ \tau$The maximum values sought are above the norm in this case established for Mexico City.

For Daily Ozone Maximums

If n = Number of data points above the surplus and $\tau=\sigma$the average data points for that year, with $\sigma = media\ size$ N

$$\theta|X\sim NB\left(\frac{\sigma^2}{\frac{N^2}{n}+\sigma^2}*\bar{x}+\frac{\frac{N^2}{n}}{\frac{N^2}{n}+\sigma^2}*\mu,\frac{\frac{N^2}{n}*\sigma^2}{\frac{N^2}{n}+\sigma^2}\right)$$

$$\sim\frac{1}{N^2+n\sigma^2}(n(\sigma^2\bar{x}+N^2\mu),\mu N^2\sigma^2)$$

$$\theta|X\ \sim\ N(a',b^{2'})$$

a function with a maximum length of the average data with maxXi

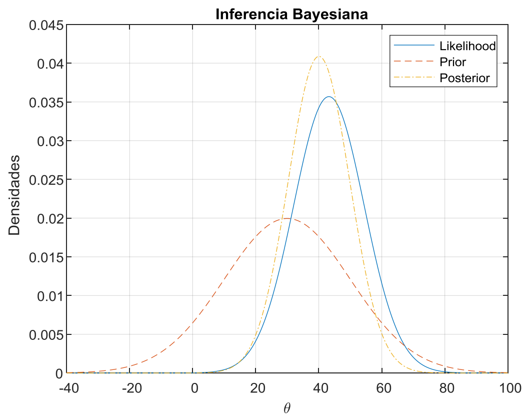
We introduce a Gaussian random number of size n

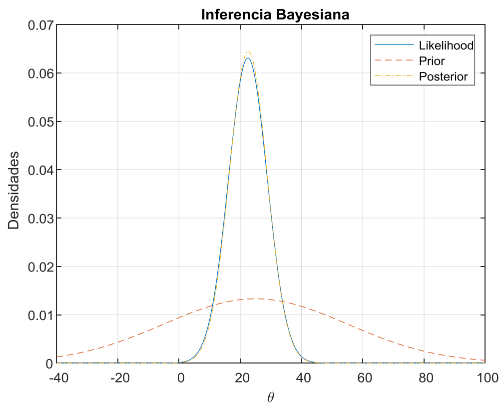
$$\varphi(maxXi) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{(maxXi\ )^2}{2\sigma^2}\right)}$$

$$\theta|X \sim NB\left(\frac{n^2}{\frac{\sigma^2}{n}+n^2} * \overline{maxXi} + \frac{\frac{\sigma^2}{n}}{\frac{\sigma^2}{n}+n^2} * \mu, \frac{\frac{\sigma^2}{n} * n^2}{\frac{\sigma^2}{n}+n^2}\right)$$

$$\sim \frac{1}{1+n}(n\,\overline{maxXi} + \mu, \sigma^2)$$

When used in the classic way

	n = 20;
	sigma = 50;
	x = normrnd(10,sigma ,n,1);
	mu = 30;
	tau = 20;
	theta = linspace (-40, 100, 500);
	y1 = normpdf (mean(x), theta ,sigma / sqrt (n));
	y2 = normpdf (theta,mu ,tau);
	postMean = tau^2*mean(x)/(tau^2+sigma^2/n) + sigma^2*mu/n/(tau^2+sigma^2/n);
	postSD = sqrt (tau^2*sigma^2/n/(tau^2+sigma^2/n));
	y3 = normpdf (theta, postMean,postSD);
	plot (theta,y1, '-' , theta,y2, '--' , theta,y3, '-.')
	legend('Likelihood' , 'Prior' , 'Posterior')
	xlabel ('\theta')

	n = 10;
	sigma = 20;
	x = normrnd (10,sigma ,n,1);
	mu = 25;
	tau = 30;

To recall and summarize the use of the method, we have the following:
150 ppb threshold was exceeded in the year, then the **Likelihood is not just "a random sample "** , but is the **"Distribution of Critical Events"** .
So that's how we make an inference about risk, not about the average.

By using n as the exceedance count, the model is performing an **update** . It is adjusting the historical belief (the a priori information, which is the entire year) with the evidence of events that exceed the established threshold.

If n increases (more days above 150), **Posteriori** will be pulled more strongly to the right, indicating that the "air health status in Mexico City is worsening."

Using the known total variance of the series is a very sound conservative decision. It means that, even when analyzing only those n critical events, the volatility of the ozone phenomenon is recognized throughout the year, as can be seen in the daily and annual records. This prevents the model from overreacting to a single day of contingency, requiring a consistent trend before moving the subsequent mean.

The priori information is the base behavior of the city's time series, and the **n events** are the anomalies.

For example, if next year the number of events were 15 or more instead of 5, the subsequent average would rise significantly, giving us a clear mathematical metric of how much it increased and the public health risk compared to the base year (2010 for example or since the measurement year).

If the **Posteriori** deviates significantly from the **Priori** , it means that the events of that year were not "isolated peaks," but rather that there was a change in the pollution structure that the Priori alone cannot explain.

Using Bayes ' Theorem for **environmental monitoring is an alternative approach** , because it does not treat all data equally, but rather gives a "larger focus" to the days when the health of the population was at risk.

Looking at the complete system (Bayesian Inference where n are the contingency days), we have as follows.

- **The Priori PDF :** By using the entire year as the Priori, the "normal climate" of Mexico City is being defined. A single day of intense sunshine and low wind (a high O3 event) is not allowed to drastically change the perception of the problem.
- **The Weight of Contingency (n values):** By using only the data that exceed 150 ppb (threshold established in Mexico City regarding certain hours) for Likelihood, the model is being forced to ask itself: *"Are these n days statistical accidents or are they evidence that the real average is rising?"* .
- **Bayesian equilibrium :** If in 2011, for example , n had been greater than 20 , the blue curve (Posteriori) would shift sharply to the right. The current model says: *"Since there were only n days out of 365, the risk remains under control, but the Posteriori curve has already shifted slightly to warn us* . "

If only the simple average of the n days were used, the resulting mean would be unnecessarily high and alarming. If the average for the entire year were used, the danger of those n days, if few in number, would be ignored and perhaps considered outliers . Thus, the Bayesian model **finds the right balance** : it acknowledges the danger but places it within the context of the entire year, emphasizing those days with the pollution spike(s).

```
%% BAYESIAN INFERENCE
n = events above the norm;
tau = sample standard deviation
sigma = mean size
x = norm ; the adjusted pdf
mu = mean;
theta = linspace ( - lim smaller, upper limit , vector size);
y1 = normpdf (mean(x ), theta ,sigma / sqrt (n));
y2 = normpdf( theta,mu ,tau );
postMean = tau^2*mean(x)/(tau^2+sigma^2/n) + sigma^2*mu/n/(tau^2+sigma^2/n);
postSD = sqrt (tau^2*sigma^2/n/(tau^2+sigma^2/n));
y3 = normpdf( theta, postMean,postSD );
figure
plot ( theta,y1, '-' , theta,y2, '--' , theta,y3, '-.' )
title ( 'Bayesian Inference Daily Maxima [O3] ppb 2019' )
legend ( ' Probability' , 'Priori' , 'Posteriori ' )
xlabel ( '\Theta = Daily Maximum Concentrations [O3] 2019 CDMX' )
ylabel ( 'Absolute Frequencies' )
grid on
```

Normal model with unknown mean and variance

Thus, the Likelihood is assumed to be a normal distribution, $x \sim N(\mu, \sigma^2)$then the conjugate prior distribution (Normal – Inverse – Gamma) is used where the mean conditioned on the variance is Normal and the variance follows an Inverse Gamma distribution . Then, the posterior distribution is from the same family as the conjugate distribution to better obtain the mean and variance. Thus , the posterior mean becomes a weighted average between the sample mean and the prior mean, while the posterior variance combines the sample variance with the uncertainty of the prior distribution .

These models are fundamental in Bayesian statistics, as they allow us to model uncertainty in both the central tendency and the dispersion of the observed data.

Classic Bayesian model of this model

Basic equations, Likelihood is

$$L(\mu,\sigma^2,X_n) = f(X_n|\mu,\sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} e^{\left[-\frac{1}{2\pi\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right]}$$

The joint posterior distribution of mu and sigma is

$$\pi(\mu,\sigma^2|X_n)\ \alpha\ \pi(\mu,\sigma^2)L(\mu,\sigma^2,X_n)\ \alpha\ \left(\frac{1}{\sigma^2}\right)\left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} e^{\left[-\frac{1}{2\pi\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right]}$$

With the sample quasi-variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

So

$$\pi(\mu,\sigma^2|X_n)\ \alpha\ \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}+1} e^{\left[-\frac{(n-1)s^2}{2\sigma^2}\right]}\left(\frac{1}{\sigma^2}\right)^{\frac{1}{2}} e^{\left[-\frac{1}{2}\frac{(\mu-\bar{x})^2}{\sigma^2/n}\right]}$$
$$\alpha\ \pi(\sigma^2|s^2) * \pi(\mu|\sigma^2,\bar{x})$$

Being conditioned by the sample variance and the arithmetic mean of x

The marginal distribution of$\pi(\mu|X_n)$

$$\alpha\int_0^\infty\left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}+1} e^{\left[-\frac{(n-1)s^2+n(\bar{x}-\mu)^2}{2\sigma^2}\right]}d\sigma^2$$

Arranging

$a = \frac{(n-1)s^2+n(\bar{x}-\mu)^2}{2}andp = \frac{n}{2}$

$$\alpha\int_0^\infty\left(\frac{1}{\sigma^2}\right)^{p+1} e^{\left[-\frac{a}{\sigma^2}\right]}d\sigma^2$$

Which is the Reverse Range PDF

$$\int_0^\infty\left(\frac{1}{x}\right)^{p+1} e^{\left[-\frac{a}{x}\right]}dx = \frac{\Gamma(p)}{a^p}$$

Student 's t

$$\pi(t|\bar{x},s)\ \alpha\ \pi(\mu(t))|\frac{d\mu(t)}{dt})$$

$$\alpha\left[(n-1)s^2+n\left(\frac{ts}{\sqrt{n}}\right)^2\right]^{-\frac{n}{2}}\frac{s}{\sqrt{n}}\ \alpha\left[1+\frac{t^2}{n-1}\right]^{-\frac{n}{2}}$$

This being the kernel of the Student 's t density pdf , with n-1 degrees of freedom

$$(\mu|X_n) = t_{n-1}\left(\bar{x},\frac{s^2}{n}\right)$$

With the posterior predictive PDF as

$$X_{n+1} | \, x \; = \; t_{n-1} \left(\bar{x}, s^2 \left(1 + \frac{1}{n} \right) \right)$$

Then n is the number of data points in the time series, the mean of the data being $\bar{x}$and $s^2$the variance of the time series

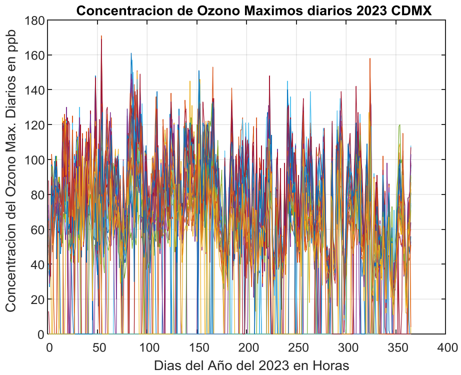
$$f(\mu | \, data) \; \alpha \; \left(1 + \frac{1}{n-1} \left(\frac{\mu - \bar{x}}{\frac{s}{\sqrt{n}}} \right)^2 \right)^{-\frac{n}{2}}$$

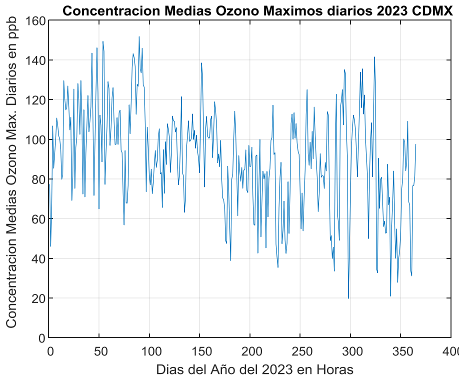
In Matlab code

```
Bayesian Analysis: Normal Model with Unknown Mean and Variance
data_O3 = Media;
% Statistics
n = length (data_O3);
x_bar = mean(data_O3);
s2 = var (data_O3);
std_error = sqrt( s2 / n );
df = n - 1; % Degrees of freedom

% Parameter Inference (Model Results)
% A posteriori mean (E[ mu|y ] = x_bar )
% Posterior variance (E[sigma^2|y] = ( df * s2) / ( df - 2)
estimated_average = x_bar ;
estimated_variance = ( df * s2) / ( df - 2);
estimated_sd_deviation = sqrt ( estimated_variance );

fprintf ( '--- Bayesian Model Results ---\n' );
fprintf ( 'Estimated Mean ( Bayes ): %.4f\n' , estimated_mean );
fprintf ( 'Estimated Variance ( Bayes ): %.4f\n' , estimated_variance );
fprintf ( ' Estimated S Deviation ( Bayes ): %.4f\n' , estimated_sd_deviation );
fprintf ( 'Degrees of freedom (n-1): %d\n\n' , df );
```

The following examples will only be given for the Time Series of 2023, 2024 and 2025

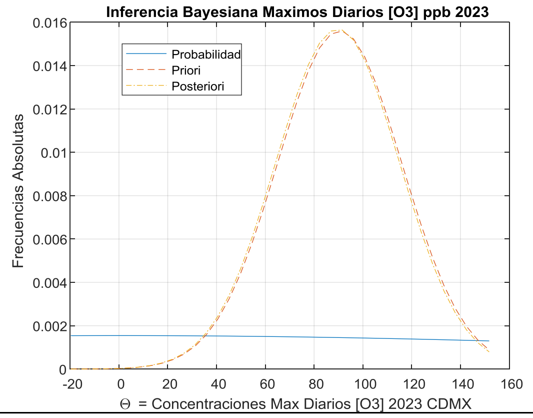
Example with daily maximum ozone concentration data O3 2023





These are the results using the modified Bayesian Inference Model with

Postmean = 89.6 ppb
PostSD = 25.43ppb



--- Bayesian Model Results ---

Estimated Mean (Bayes): 90.4918 Estimated Variance (Bayes): 656.8403 Estimated S Deviation (Bayes): 25.6289 Degrees of freedom (n-1): 364	--- Fit Statistics --- MSE: 0.000073 RMSE: 0.008561 R^2: -1.3512

The examples consist of the following figures: the time series of the annual daily O3 maximums, the time series of the mean used to estimate the unknown mean, the graph of the Modified Bayesian Inference with the calculated mean and variance values, the calculation and results using the classical method with the Student 's t-test and its fit in the box, and the results of the classical fit of the Student 's t -test in the box .

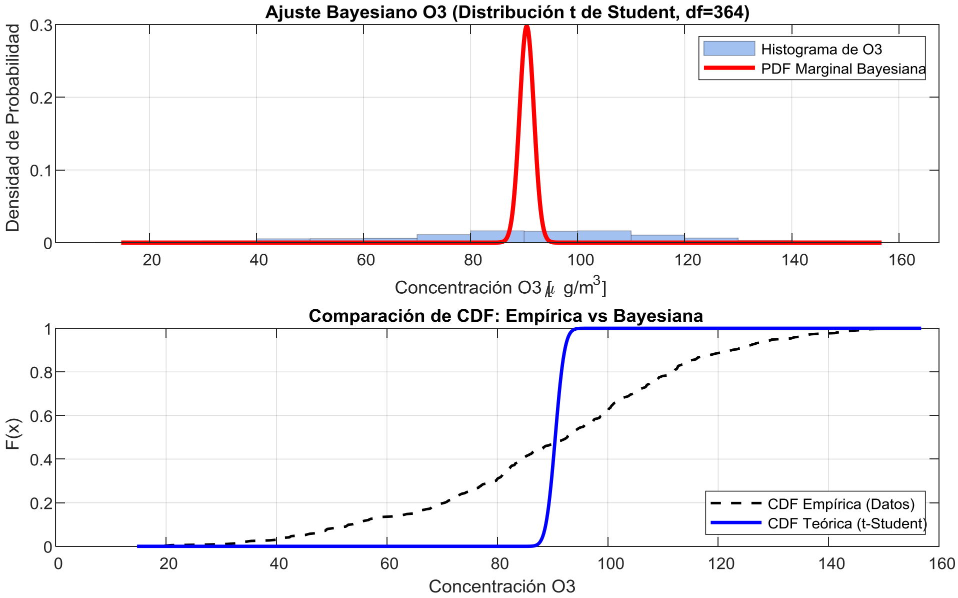


Figure 15

--- Fit Statistics --- Student Location (mu): 90.4911 Scale (sigma): 25.5233 Degrees of freedom (nu): 4027853.0758 Theoretical mean: 90.4911 Theoretical variance: 651.4372	--- Fit Metrics --- Student MSE: 0.000002 RMSE: 0.001451 R^2: 0.9325 KS test: h=0, p=0.5239

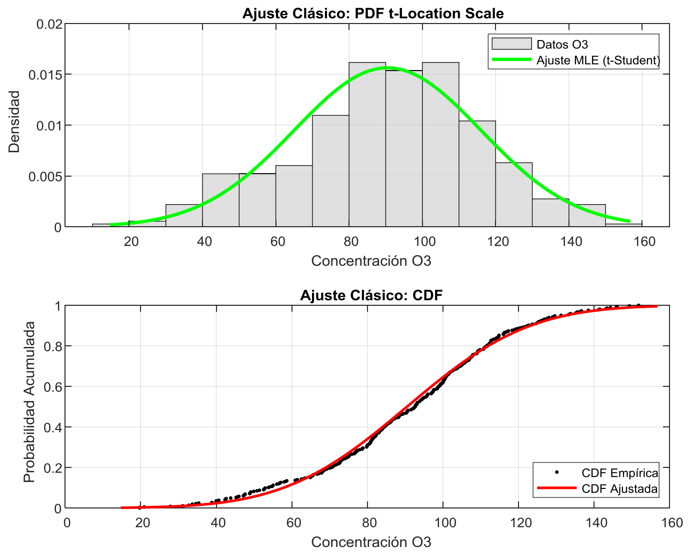


Figure 16

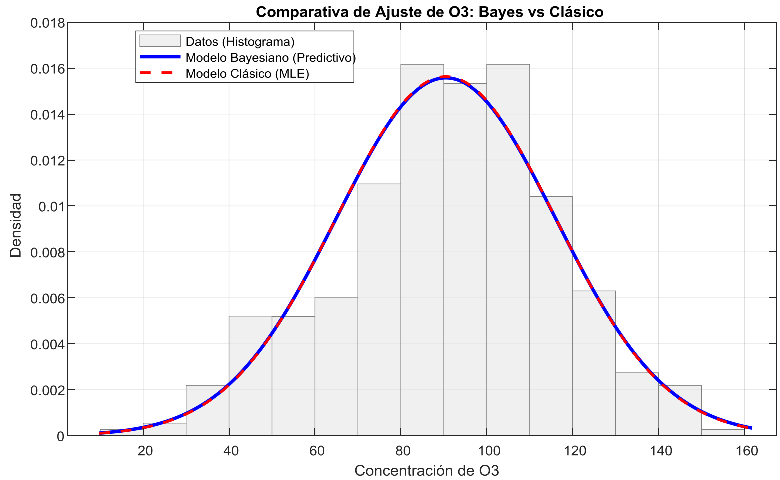
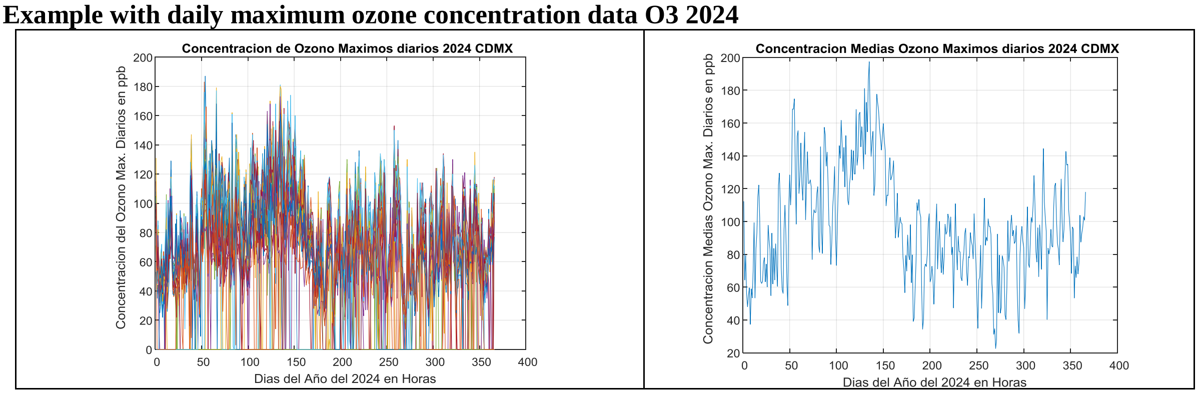


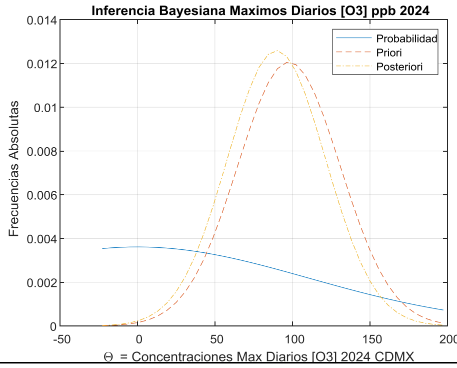
Figure 17

--- MODEL COMPARISON ---

Metrics	Bayesian	Classic
Media (mu)	90.4918	90.4911
G. Freedom	364,000	4027853.0758
RMSE	0.001450	0.001451
R^2	0.9325	0.9325

Example with daily maximum ozone concentration data O3 2024



	These are the results using the modified Bayesian Inference Model with Postmean = 89.68ppb PostSD = 31.68ppb The mean is ME = 2*Me

--- Bayesian Model Results ---

Estimated Mean (Bayes): 97.7440 Estimated Variance (Bayes): 1100.4958 Estimated S Deviation (Bayes): 33.1737 Degrees of freedom (n-1): 365	--- Fit Statistics --- MSE: 0.000048 RMSE: 0.006895 R^2: -1.8439

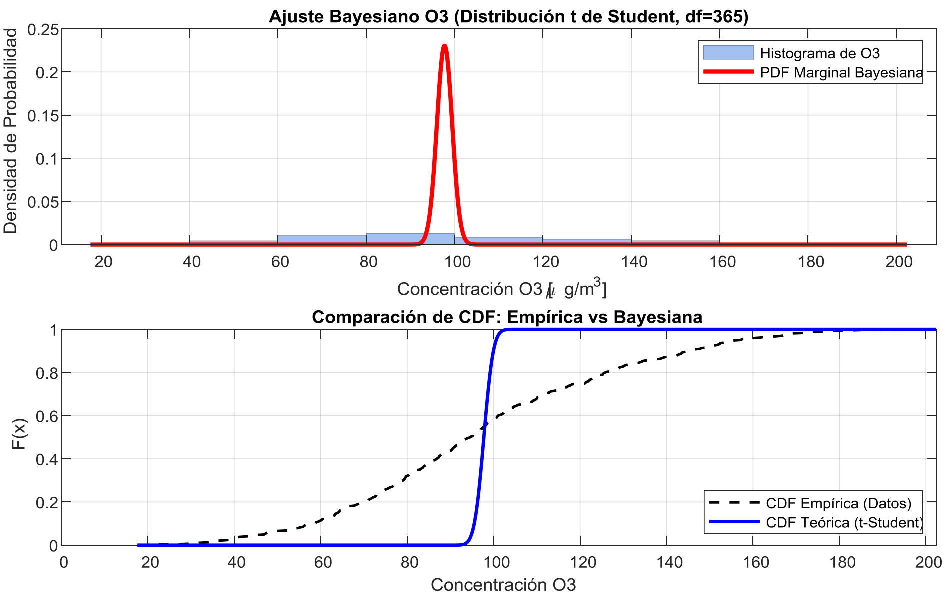


Figure 18

--- Fit Statistics --- Student Location (mu): 97.7450 Scale (sigma): 33.0393 Degrees of freedom (nu): 6126954.6279 Theoretical mean: 97.7450 Theoretical variance: 1091.5949	--- Fit Metrics --- Student MSE: 0.000002 RMSE: 0.001397 R^2: 0.8833 KS test: h=0, p=0.0725

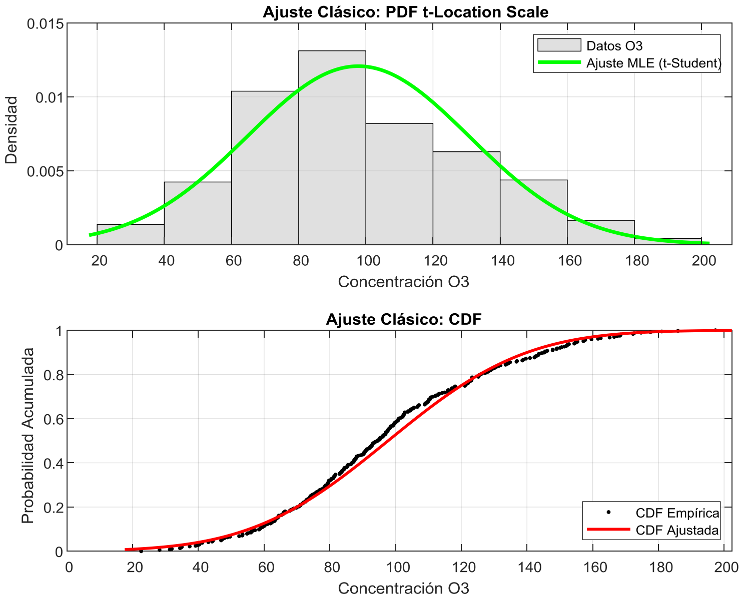


Figure 19

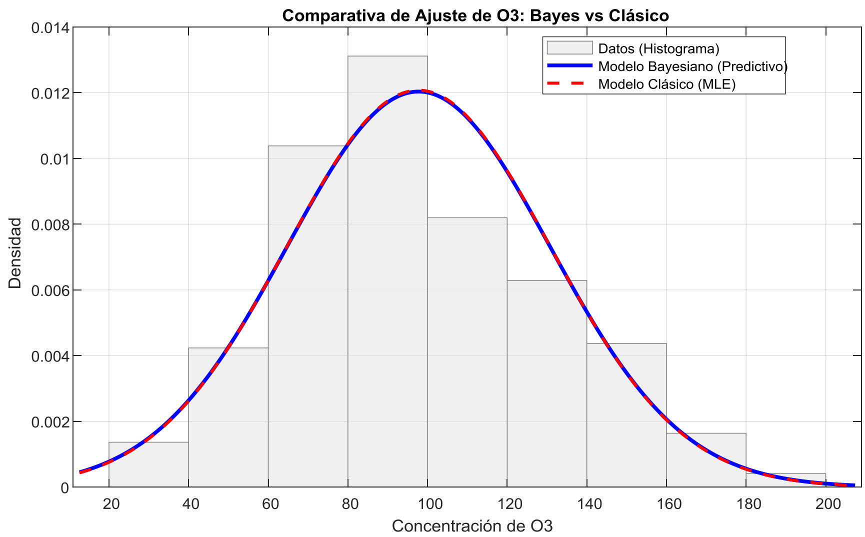
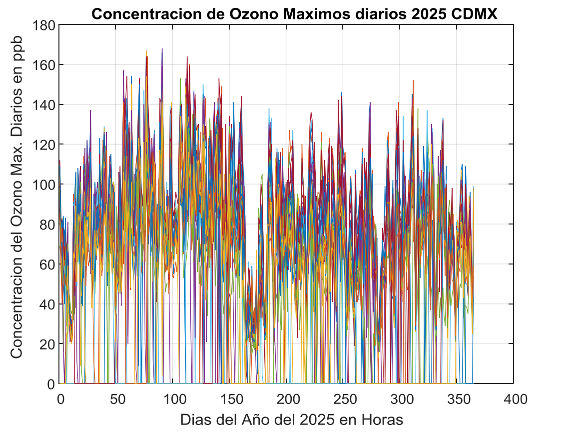


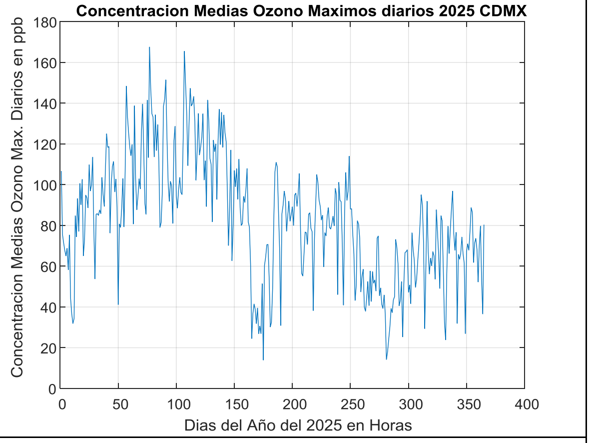
Figure 20

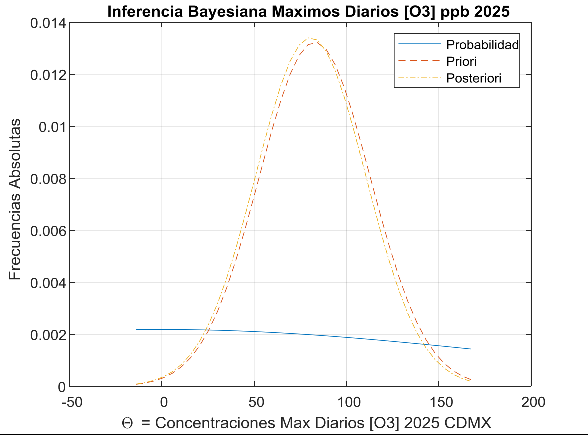
--- MODEL COMPARISON ---

Metrics	Bayesian	Classic
Media (mu)	97.7440	97.7450
G. Freedom	364,000	6126954.6279
RMSE	0.001392	0.001397
R^2	0.8841	0.8833

Example with daily maximum ozone concentration data O3 2025







These are the results using the modified Bayesian Inference Model with

Postmean = 80.61 ppb
PostSD = 29.7 ppb

--- Bayesian Model Results ---

Estimated Mean (Bayes): 82.8145 Estimated Variance (Bayes): 913.9965 Estimated S Deviation (Bayes): 30.2324 Degrees of freedom (n-1): 364	--- Fit Statistics --- MSE: 0.000450 RMSE: 0.021210 R^2: -20.8407

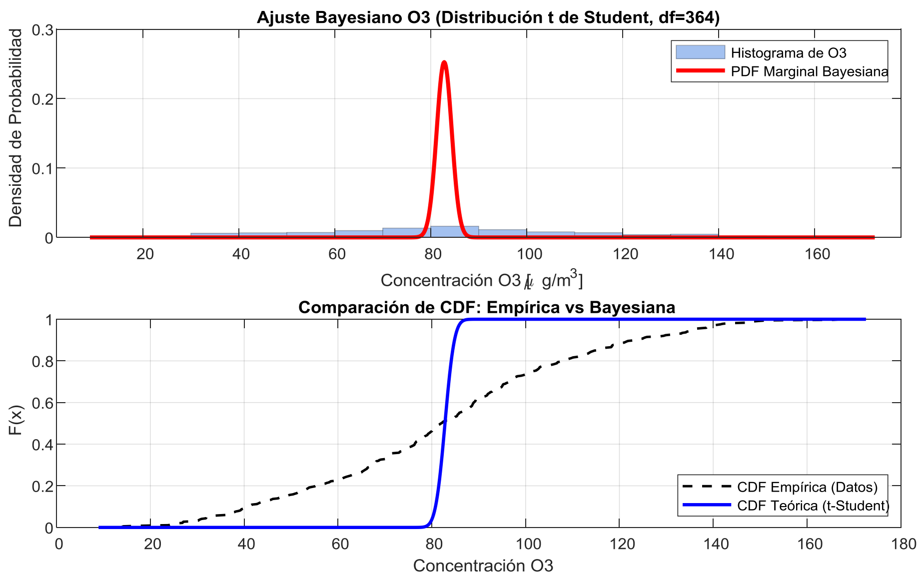


Figure 21

--- Fit Statistics --- Student Location (mu): 82.8148 Scale (sigma): 30.1080 Degrees of freedom (nu): 2413608.6024 Theoretical mean: 82.8148 Theoretical variance: 906.4925	--- Fit Metrics --- Student MSE: 0.000002 RMSE: 0.001342 R^2: 0.9125 KS test: h=0, p=0.7843

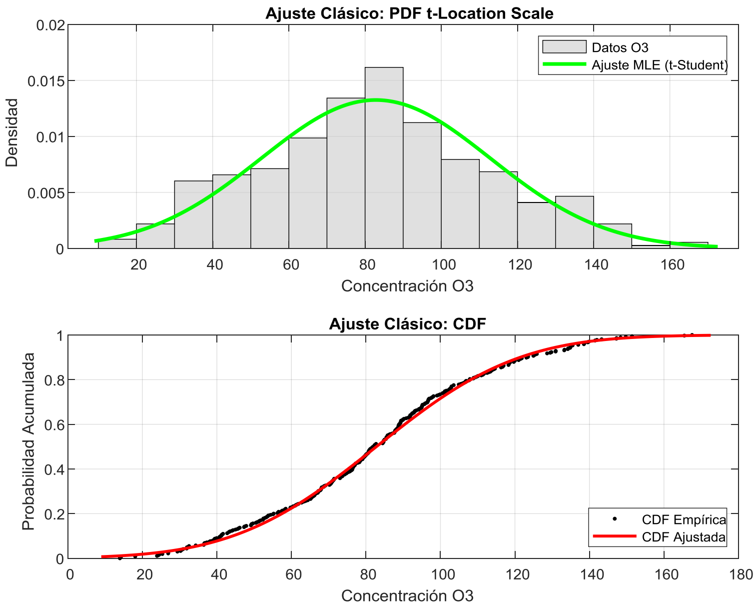


Figure 22

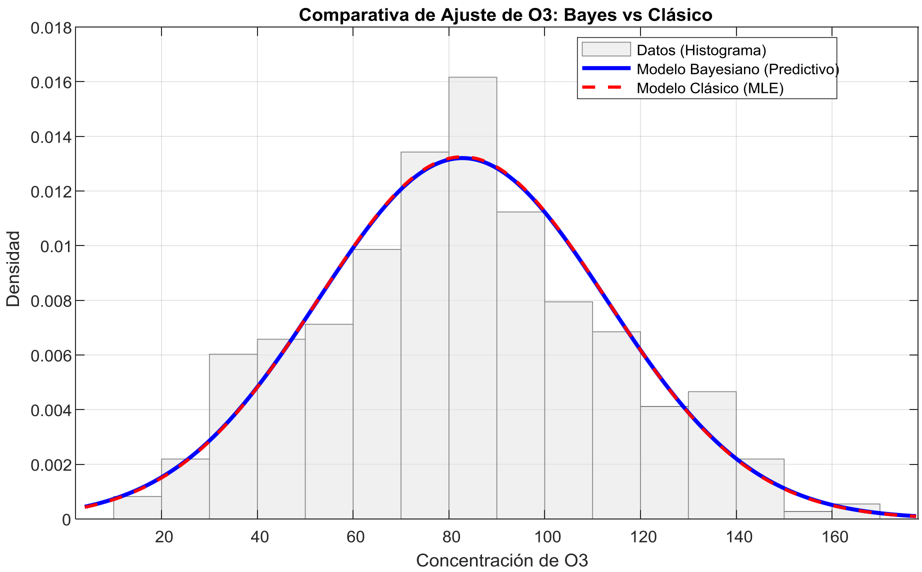


Figure 23

-- MODEL COMPARISON ---

Metrics	Bayesian	Classic
Media (mu)	82.8145	82.8148
G. Freedom	364,000	2413608.6024
RMSE	0.001339	0.001342
R^2	0.9129	0.9125

Conclusions

The proposed Bayesian Inference model for unknown mean and variance is shown to converge almost exactly with the results of classical statistics based on the Student 's t-distribution. The estimated mean and R^2 values (greater than 0.90 in the analyzed cases from 2023-2025) validate the algorithm's accuracy for the analysis of complex time series.

The model offers a robust rationale for environmental monitoring by giving a broader focus to contingency days without ignoring the annual context. This avoids the bias of simple averages, recognizing the danger of pollution spikes (O3 > 150 ppb) without generating unnecessary alarm based on *outliers* or isolated events.

The shift of the *posterior distribution* relative to the *prior distribution* acts as a mathematical indicator of structural changes in air quality. An increase in the number of critical events (n) shifts the posterior mean to the right, providing a clear metric of the increase in public health risk compared to baseline years.

Multivariate Applicability: Although the study focuses on ozone, the methodology is perfectly transferable to other regulated pollutants such as PM10 and PM2.5, or even to climatic variables such as maximum temperatures, proving to be a versatile tool for management and monitoring.

Degrees of Freedom Treatment, it is observed that, while the Bayesian model uses degrees of freedom based on the sample size (n-1), the classical fit can yield extremely highdegrees of freedom (nu) values, indicating that the distribution of ozone concentrations tends to behave in a Gaussian manner in large samples.

Robustness in Variance Estimation: The relationship between the Inverse-Gamma distribution and the Student's t allows the estimated variance to integrate both the simple variance and the uncertainty of the prior information, resulting in a more reliable dispersion metric for assessing environmental risk.

In memory of my mother Ana María Jiménez Castellanos (November 24, 2025), dedicating this work to her.

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