



# Analysis of the solution of nth-order nonlinear mixed partial differential equations using the Adomian decomposition method and the ZJ transform

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**Abstract:** In this new article, the Adomian Decomposition Method is presented, along with the application of the ZJ transform together for the solution of nonlinear Mixed PDEs of order n with p+q =n, where n is a positive integer, with good effectiveness and potential of this hybrid approach to obtain approximate analytical solutions of nonlinear problems.

**Keywords:** ZJ Transform, Adomian Decomposition Method, Adomian Polynomials, Nonlinear Differential Equations, Series, Mixed Nonlinear Partial Differential Equations.

### Introduction:

We begin our study of nth-order mixed nonlinear partial differential equations (DDEs), which also model a vast range of complex phenomena in various scientific and engineering disciplines. However, the inherently nonlinear nature of these equations often makes obtaining exact analytical solutions difficult.

Among the analytical techniques for addressing ODEs as well as nonlinear PDEs, the **Adomian Decomposition Method (ADM)** has emerged as a versatile and effective tool. Introduced by George Adomian in the 1980s, the ADM offers a methodology for obtaining solutions in the form of convergent series without requiring linearization, discretization, or the introduction of small perturbation parameters—features that often limit the applicability of other traditional methods.

Integral transforms convert a differential equation into an algebraic equation in the transform domain, which is often easier to solve. Subsequently, applying the inverse transform allows obtaining the solution in the original domain. The ZJ transform will be applied again under these conditions, and the strategic combination of the Adomian Decomposition Method with integral transform techniques will be used to solve them. The Adomian Decomposition Method handles the nonlinear part by generating the corresponding Adomian polynomials.

### nth Order Nonlinear Mixed PDEs

Let us consider a nonlinear PDE of nth order of the general form:

$$L(u(x, t)) + R(u(x, t)) + N(u(x, t)) = h(x, t)$$

where:

- $L(u(x, t)) = \frac{\partial^n}{\partial x^p \partial y^q}$  the nth order linear operator with p + q = n, p, q are positive integers
- $R(u(x, t))$  represents the linear operator with first-order derivatives.
- $N(u(x, t))$  represents the nonlinear operator.
- $h(x, t)$  is the non-homogeneous function (source term).

**Thus, the EDP is:**

$$\frac{\partial^n u(x, t)}{\partial x^p \partial y^q} + R(u(x, t)) + N(u(x, t)) = h(x, t)$$

With  $U = \frac{\partial^q u}{\partial y^q}$

$$\frac{\partial^p U}{\partial x^p} + R(u(x, t)) + N(u(x, t)) = h(x, t)$$

Now, taking the property of the p- th derivative of the ZJ Transform on both sides of the equation with respect to x, the initial conditions at x = 0 of fy, and then taking the inverse ZJ Transform at x, we can write the PDE in the following form



$$\frac{\partial^q u(x, t)}{\partial y^q} = \phi(x, y) + ZJ_x^{-1} \left[ \left( \frac{\beta}{z} \right)^p ZJ_x [h(x, t) - R(u(x, t)) - N(u(x, t))] \right]$$

With  $\phi(x, y)$  the function that incorporates the boundary conditions given at  $x = 0$  in the derivatives

Now, taking the property of the  $p$ -th derivative of the ZJ Transform on both sides of the equation with respect to  $y$  and the initial conditions at  $y = 0$  of  $y$ , then taking the inverse ZJ Transform on  $y$ , we can write the PDE in the following form

$$u(x, y) = a(x, y) + ZJ_y^{-1} \left[ \left( \frac{\beta}{z} \right)^q ZJ_y \left[ ZJ_x^{-1} \left[ \left( \frac{\beta}{z} \right)^p ZJ_x [h(x, t) - R(u(x, t)) - N(u(x, t))] \right] \right] \right]$$

With  $a(x, y)$  part of the solution of the transformation that satisfies all linear initial conditions, now the iterative scheme by integrating the Adomian method in the nonlinear part has

$$u_0(x, y) = a(x, y) + ZJ_y^{-1} \left[ \left( \frac{\beta}{z} \right)^q ZJ_y \left[ ZJ_x^{-1} \left[ \left( \frac{\beta}{z} \right)^p ZJ_x [h(x, t)] \right] \right] \right]$$

$$u_{n+1}(x, y) = ZJ_y^{-1} \left[ \left( \frac{\beta}{z} \right)^q ZJ_y \left[ ZJ_x^{-1} \left[ \left( \frac{\beta}{z} \right)^p ZJ_x [R(u(x, t)) - N(u(x, t))] \right] \right] \right]$$

$$Nu(x, t) = \sum_{n=0}^{\infty} A_n(x, t)$$

### Application of the Adomian Decomposition Method

Now, we apply the MDA to solve the equation in the ZJ domain. We assume that the solution  $u(t)$  can be expressed as an infinite series. Superposition principle, the solution can be represented as an infinite series  $\sum_{n=0}^{\infty} \hat{\phi}_n(x, t)$

If we observe the nonlinear operator  $Nu(x, t)$  from our equation, we decompose it as a series of Adomian polynomials as  $Nu(x, t) = \sum_{n=0}^{\infty} A_n(x, t)$  where  $A_n$  are Adomian polynomials of  $u_n$  and it can be determined by the relation

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[ N \sum_{i=0}^{\infty} \lambda^i u_i \right]$$

Now calculating the Adomian Polynomials

$$\begin{aligned} A_0 &= u_0^2 \\ A_1 &= 2u_0u_1 \\ A_2 &= 2u_0u_2 + u_1^2 \end{aligned}$$

With

$$\left( \frac{\partial u_0}{\partial y} \right)^2 = A_0$$

### Application Examples

#### Example 1

Let the Mixed Nonlinear Third Order Partial Differential Equation be

$$\frac{\partial^3 u}{\partial x \partial y^2} - \left( \frac{\partial u}{\partial y} \right)^2 + u^2 = e^y \cos(x)$$

With  $u(x, 0) = \sin(x)$   $\frac{\partial u(x, 0)}{\partial y} = \sin(x)$  and  $\frac{\partial^2 u(0, y)}{\partial y^2} = 0$

with  $U = \frac{\partial^2 u}{\partial y^2}$  applying the ZJ Transform in  $x$ , on both sides

$$ZJ_x \left[ \frac{\partial U}{\partial x} \right] = ZJ_x \left[ e^y \cos(x) + \left( \frac{\partial u}{\partial y} \right)^2 - u^2 \right]$$

Now, let's look at the properties of the first derivative of the Z-transform

$$ZJ \left[ \frac{\partial U}{\partial x} \right] = \frac{z}{\beta} \hat{\phi} - u(0, y) \frac{\beta^n}{z}$$

Replacing the boundary condition which is  $\frac{\partial^2 u(0, y)}{\partial y^2} = 0$  Integrating both times we get  $u(0, y) = 0$ , Now taking the inverse ZJ transform with respect to  $x$



$$\hat{\phi} = \frac{\beta}{z} ZJ_x[e^y \cos(x)] + \frac{\beta}{z} ZJ_x \left[ \left( \frac{\partial u}{\partial y} \right)^2 - u^2 \right]$$

$$ZJ_x^{-1}[\hat{\phi}] = ZJ_x^{-1} \left[ \frac{\beta}{z} ZJ_x[e^y \cos(x)] \right] + ZJ_x^{-1} \left[ \frac{\beta}{z} ZJ_x \left[ \left( \frac{\partial u}{\partial y} \right)^2 - u^2 \right] \right]$$

$$\frac{\partial^2 u(x, y)}{\partial y^2} = e^y \text{sen}(x) + ZJ_x^{-1} \left[ \frac{\beta}{z} ZJ_x \left[ \left( \frac{\partial u}{\partial y} \right)^2 - u^2 \right] \right]$$

Now taking the ZJ transform with respect to a

$$ZJ_y \left[ \frac{\partial^2 u(x, y)}{\partial y^2} \right] = ZJ_y[e^y \text{sen}(x)] + ZJ_y \left[ ZJ_x^{-1} \left[ \frac{\beta}{z} ZJ_x \left[ \left( \frac{\partial u}{\partial y} \right)^2 - u^2 \right] \right] \right]$$

Taking the property of the second derived from the ZJ transform and  $u(x, 0) = \text{sen}(x) \frac{\partial u(x, 0)}{\partial y} = \text{sen}(x)$

$$ZJ_y \left[ \frac{\partial^2 u(x, y)}{\partial y^2} \right] = \frac{z^2}{\beta^2} \hat{\phi} - \frac{\beta^n}{\beta} u(x, 0) - \frac{\beta^n}{z} \frac{\partial u(x, 0)}{\partial y}$$

It is held

$$\hat{\phi}_y = \left( \frac{\beta^{n+1}}{z^2} - \frac{\beta^{n+2}}{z^3} \right) \text{sen}(x) + \frac{\beta^2}{z^2} ZJ_y[e^y] \text{sen}(x) + ZJ_y \left[ ZJ_x^{-1} \left[ \frac{\beta}{z} ZJ_x \left[ \left( \frac{\partial u}{\partial y} \right)^2 - u^2 \right] \right] \right]$$

Now simplifying the linear terms and taking the inverse ZJ transform in y

$$ZJ_y^{-1}[\hat{\phi}_y] = ZJ_y^{-1} \left[ \left( \frac{\beta^{n+1}}{z^2} - \frac{\beta^{n+2}}{z^3} \right) \text{sen}(x) \right] + ZJ_y^{-1} \left[ \frac{\beta^2}{z^2} ZJ_y[e^y] \text{sen}(x) \right] + ZJ_y^{-1} \left[ ZJ_y \left[ ZJ_x^{-1} \left[ \frac{\beta}{z} ZJ_x \left[ \left( \frac{\partial u}{\partial y} \right)^2 - u^2 \right] \right] \right] \right]$$

$$u(x, y) = e^y \text{sen}(x) \text{ and this is } u_0$$

Now for the non-linear part

$$N_1(u) = \left( \frac{\partial u}{\partial y} \right)^2 \quad \text{and} \quad N_2(u) = u^2$$

$$A_0 = \left( \frac{\partial u_0}{\partial y} \right)^2 = (e^y \text{sen}(x))^2 \quad \text{and} \quad B_0 = (u_0)^2 = (e^y \text{sen}(x))^2$$

Thus the recursive relation is

$$u_0(x, y) = e^y \text{sen}(x)$$

$$u(x, y)_{n+1} = ZJ_y^{-1} \left[ \left( \frac{\beta^2}{z^2} \right) ZJ_y \left[ ZJ_x^{-1} \left[ \frac{\beta}{z} ZJ_x \left[ \sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n \right] \right] \right] \right]$$

By calculating the first term,  $u(x, y)_1 = 0$  we can see that they cancel out, thus truncating the series and giving the solution



$$u(x, y) = e^y \text{sen}(x)$$

### Example 2

Let the Mixed Nonlinear Fourth Order Partial Differential Equation be

$$\frac{\partial^4 u}{\partial x^2 \partial y^2} + u \left( \frac{\partial u}{\partial y} \right) - u^2 = 0$$

$$\text{With } u(x, 0) = x \frac{\partial u(x, 0)}{\partial y} = x \frac{\partial^2 u(0, y)}{\partial y^2} = 0 \quad \frac{\partial^3 u(0, y)}{\partial x \partial y^2} = e^y$$

by  $U = \frac{\partial^2 u}{\partial y^2}$  applying the ZJ Transform in x, on both sides

$$ZJ_x \left[ \frac{\partial^2 U}{\partial x^2} \right] = ZJ_x \left[ u^2 - u \left( \frac{\partial u}{\partial y} \right) \right]$$

Now, regarding the properties of the second derivative of the ZJ transform

$$ZJ \left[ \frac{\partial^2 U}{\partial y^2} \right] = \frac{z^2}{\beta^2} \hat{\varphi} - \frac{\beta^n}{\beta} u(x, 0) - \frac{\beta^n}{z} u'(x, 0)$$

In this case, it is at  $x=0$ ,  $ZJ \left[ \frac{\partial^2 U}{\partial x^2} \right] = \frac{z^2}{\beta^2} \hat{\varphi} - \frac{\beta^n}{\beta} u(0, y) - \frac{\beta^n}{z} u'(0, y)$ , so integrating the BC  $\frac{\partial^2 u(0, y)}{\partial y^2} = 0$  gives us

$u(0, y) = 0$  and now  $\frac{\partial^3 u(0, y)}{\partial x \partial y^2} = \frac{\partial U(0, y)}{\partial x} = e^y$  we have:

$$\hat{\varphi}_x = \frac{\beta^{n+2}}{z^3} e^y + \frac{\beta^2}{z^2} ZJ_x \left[ u^2 - u \left( \frac{\partial u}{\partial y} \right) \right]$$

Applying the inverse ZJ transform on x gives us

$$\frac{\partial^2 u}{\partial y^2} = x e^y + ZJ_x^{-1} \left[ \frac{\beta^2}{z^2} ZJ_x \left[ u^2 - u \left( \frac{\partial u}{\partial y} \right) \right] \right]$$

Now, taking the ZJ transform in y, using the same property of the second derivative and the BC, we have,

$u(x, 0) = x$ ,  $\frac{\partial u(x, 0)}{\partial y} = x$  and taking the inverse ZJ transform in y, the linear and nonlinear terms

$$u(x, y) = ZJ_y^{-1} \left[ x \left( \frac{\beta^{n+1}}{z^2} + \frac{\beta^{n+2}}{z^3} \right) + \frac{\beta^2}{z^2} x ZJ_y [e^y] \right] = x e^y$$

Applying Adomian's method with  $u_0 = x e^y$

$$N(u) = u^2 - u \left( \frac{\partial u}{\partial y} \right)$$

With the first term  $A_0 = u_0^2 - u_0 \left( \frac{\partial u_0}{\partial y} \right) = 0$ , it is immediately truncated, therefore the solution is

$$u(x, y) = x e^y$$

### Example 3

Let the Fifth Order Mixed Nonlinear Partial Differential Equation be

$$\frac{\partial^5 u}{\partial x^3 \partial y^2} - u \left( \frac{\partial u}{\partial y} \right) + u^2 = e^x \cos(y)$$

$$\text{With } u(0, y) = -\cos(y) \quad \frac{\partial u(0, y)}{\partial x} = -\cos(y) \quad \frac{\partial^2 u(0, y)}{\partial x^2} = -\cos(y) \quad \frac{\partial^3 u(x, 0)}{\partial x^3} = -e^x$$

by  $U = \frac{\partial^3 u}{\partial x^3}$  applying the ZJ Transform on y, on both sides

$$ZJ_y \left[ \frac{\partial^2 U}{\partial y^2} \right] = ZJ_y [e^x \cos(y)] + ZJ_y \left[ u \left( \frac{\partial u}{\partial y} \right) - u^2 \right]$$

Now, regarding the properties of the second derivative of the ZJ transform

$$ZJ \left[ \frac{\partial^2 U}{\partial y^2} \right] = \frac{z^2}{\beta^2} \hat{\varphi} - \frac{\beta^n}{\beta} u(x, 0) - \frac{\beta^n}{z} u'(x, 0)$$

In this case it is at  $y=0$ ,  $ZJ \left[ \frac{\partial^2 U}{\partial y^2} \right] = \frac{z^2}{\beta^2} \hat{\varphi} - \frac{\beta^n}{\beta} u(x, 0) - \frac{\beta^n}{z} u'(x, 0)$ , so integrating the BC  $\frac{\partial^3 u(x, 0)}{\partial x^3} = -e^x$  gives us

$u(x, 0) = -e^x$  and now  $\frac{\partial u(x, 0)}{\partial y} = 0$  We don't have it, so we're left with:

$$\hat{\varphi}_y = \frac{\beta^2}{z^2} e^x ZJ_y [\cos(y)] - \frac{\beta^{n+1}}{z^2} e^x + \frac{\beta^2}{z^2} ZJ_y \left[ u \left( \frac{\partial u}{\partial y} \right) - u^2 \right]$$

Now the inverse ZJ transform in y



$$\frac{\partial^3 u}{\partial x^3} = ZJ_y^{-1} \left[ \frac{\beta^2}{z^2} e^x ZJ_y [\cos^2(y)] - \frac{\beta^{n+1}}{z^2} e^x \right] + ZJ_y^{-1} \left[ \frac{\beta^2}{z^2} ZJ_y \left[ u \left( \frac{\partial u}{\partial y} \right) - u^2 \right] \right]$$

Now, regarding the property of the third derivative of the ZJ transform

$$ZJ \left[ \frac{\partial^3 u(x, t)}{\partial y^3} \right] = \frac{z^3}{\beta^3} \hat{\varphi} - \frac{z\beta^n}{\beta^2} u(x, 0) - \frac{\beta^n}{\beta} u'(x, 0) - \frac{\beta^n}{z} u''(x, 0)$$

In this case, it is the ZJ transform in x

$$ZJ_x \left[ \frac{\partial^3 u(x, t)}{\partial x^3} \right] = \frac{z^3}{\beta^3} \hat{\varphi} - \frac{z\beta^n}{\beta^2} u(0, y) - \frac{\beta^n}{\beta} u'(0, y) - \frac{\beta^n}{z} u''(0, y)$$

Taking the inverse ZJ transform in x, we thus have the linear and non-linear parts

$$u(x, y) = ZJ_x^{-1} \left[ ZJ_x \left[ ZJ_y^{-1} \left[ \frac{\beta^2}{z^2} e^x ZJ_y [\cos^2(y)] - \frac{\beta^{n+1}}{z^2} e^x \right] \right] + ZJ_x^{-1} \left[ ZJ_x \left[ ZJ_y^{-1} \left[ \frac{\beta^2}{z^2} ZJ_y \left[ u \left( \frac{\partial u}{\partial y} \right) - u^2 \right] \right] \right] \right] \right]$$

Thus, solving the linear part is

$$u_0 = -e^x \cos^2(y)$$

Now, using Adomian's method in the nonlinear part, it is

$$N(u) = u \left( \frac{\partial u}{\partial y} \right) - u^2 = u_0 \left( \frac{\partial u_0}{\partial y} \right) - u_0^2 = 0 \quad \text{y asi } A_0 - B_0 = 0$$

The solution is immediately truncated upon iterating for u1; therefore, the exact solution is

$$u(x, y) = -e^x \cos^2(y)$$

#### Example 4

Let the second-order nonlinear mixed Monge-Ampere partial differential equation be nonhomogeneous

$$\left( \frac{\partial^2 u}{\partial x^2} \right) \left( \frac{\partial^2 u}{\partial y^2} \right) - \left( \frac{\partial^2 u}{\partial x \partial y} \right)^2 = f(x, y)$$

$$\text{With } u(0, y) = y^2 \frac{\partial u(0, y)}{\partial x} = y$$

Rearranging, we have and applying the ZJ transform in x

$$ZJ_x \left[ \frac{\partial^2 u}{\partial x^2} \right] = ZJ_x \left[ \frac{f(x, y) + \left( \frac{\partial^2 u}{\partial x \partial y} \right)^2}{\left( \frac{\partial^2 u}{\partial y^2} \right)} \right]$$

Now with f = 0 and the property of the second derived from the ZJ transform and substituting the BC

$$\hat{\varphi}_x = \frac{\beta^{n+1}}{z^2} y^2 + \frac{\beta^{n+2}}{z^3} y + \frac{\beta^2}{z^2} ZJ_x \left[ \frac{\left( \frac{\partial^2 u}{\partial x \partial y} \right)^2}{\left( \frac{\partial^2 u}{\partial y^2} \right)} \right]$$

Now applying the inverse ZJ transform in x we have

$$u(x, y) = ZJ_x^{-1} \left[ \frac{\beta^{n+1}}{z^2} y^2 \right] + ZJ_x^{-1} \left[ \frac{\beta^{n+2}}{z^3} y \right] + ZJ_x^{-1} \left[ ZJ_x \left[ \frac{\left( \frac{\partial^2 u}{\partial x \partial y} \right)^2}{\left( \frac{\partial^2 u}{\partial y^2} \right)} \right] \right]$$

$$u(x, y) = y^2 + xy + ZJ_x^{-1} \left[ ZJ_x \left[ \frac{\left( \frac{\partial^2 u}{\partial x \partial y} \right)^2}{\left( \frac{\partial^2 u}{\partial y^2} \right)} \right] \right]$$

$$u(x, y) = y^2 + xy + ZJ_x^{-1} [ZJ_x [N(u)]]$$

With

$$u_0 = y^2 + xy$$

Using Adomian's Method, we have



$$\frac{\partial^2 u_o}{\partial y^2} = 2 \quad \frac{\partial u_o}{\partial y} = 2y + x \quad \frac{\partial^2 u_o}{\partial x \partial y} = 1$$

$$N(u_o) = A_o$$

$$A_o = \frac{\left(\frac{\partial^2 u_o}{\partial x \partial y}\right)^2}{\left(\frac{\partial^2 u_o}{\partial y^2}\right)} = \frac{1}{2}$$

Thus  $n = 1$

$$u_1 = ZJ_x^{-1}[ZJ_x[A_o]]$$

So this is it

$$u_1 = \frac{x^2}{4}$$

$$\frac{\partial^2 u_1}{\partial y^2} = 0 \quad \frac{\partial u_1}{\partial y} = 0 \quad \frac{\partial^2 u_1}{\partial x \partial y} = 0$$

It is immediately truncated, therefore the exact solution for this homogeneous form is the following and can be verified.

$$u(x, y) = y^2 + xy + \frac{x^2}{4}$$

As can be seen, what needs to be generalized is the  $n$ th derivative of the ZJ Transformation, then, equivalently,  $y = t$ , or  $x$

$$ZJ \left[ \frac{\partial^m u(x, t)}{\partial y^m} \right] = \left(\frac{z}{\beta}\right)^m \frac{\beta^n}{z} \int_0^\infty e^{-\frac{zt}{\beta}} u(x, t) dt - \frac{\beta^n}{z} \sum_{k=0}^{m-1} \left[ \left(\frac{z}{\beta}\right)^{m-1-k} \right] \frac{\partial^k u(x, 0)}{\partial t^k}$$

$$\hat{\varphi} = \frac{\beta^n}{z} \int_0^\infty e^{-\frac{zt}{\beta}} u(x, t) dt$$

Table of some derivatives

Derivative	Expression
$ZJ \left[ \frac{\partial u(x, t)}{\partial y} \right]$	$\frac{z}{\beta} \hat{\varphi} - \frac{\beta^n}{z} u(x, 0)$
$ZJ \left[ \frac{\partial^2 u(x, t)}{\partial y^2} \right]$	$\frac{z^2}{\beta^2} \hat{\varphi} - \frac{\beta^n}{\beta} u(x, 0) - \frac{\beta^n}{z} u'(x, 0)$
$ZJ \left[ \frac{\partial^3 u(x, t)}{\partial y^3} \right]$	$\frac{z^3}{\beta^3} \hat{\varphi} - \frac{z\beta^n}{\beta^2} u(x, 0) - \frac{\beta^n}{\beta} u'(x, 0) - \frac{\beta^n}{z} u''(x, 0)$
$ZJ \left[ \frac{\partial^4 u(x, t)}{\partial y^4} \right]$	$\frac{z^4}{\beta^4} \hat{\varphi} - \frac{z^2\beta^n}{\beta^3} u(x, 0) - \frac{z\beta^n}{\beta^2} u'(x, 0) - \frac{\beta^n}{\beta} u''(x, 0) - \frac{\beta^n}{z} u'''(x, 0)$

### Conclusions

The combination of the Adomian Decomposition Method (ADM) with the ZJ transform (Adomian-ZJ method) offers several significant advantages over classical methods for solving certain nonlinear PDEs:

**However, it is important to consider some limitations, such as in the case of nonlinear PDEs:**

- The convergence of the Adomian series is not always guaranteed and may depend on the nature of the nonlinear equation and the conditions of the problem.
- Calculating Adomian polynomials for complex nonlinearities can be laborious.
- Applying the inverse ZJ transform to the components  $\hat{\varphi}_n(x, t)$  may not always be trivial and may require special techniques or the use of transform tables.

$n$ th- order mixed nonlinear PDEs offers an elegant alternative to classical methods for solving certain nonlinear PDEs of this type, especially those where linearization or perturbations are not suitable or where an approximate analytical solution is sought without resorting to discretization as in the previous article.

In memory of my mother Ana María Jiménez Castellanos (November 24, 2025), dedicating this work to her.



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