



An Optimization Model For Products with Variable Production Cost, Selling Price and Varying Distribution of Customer Demand

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Abstract: In this paper, for the products with variable production cost and selling price and different distribution of customer demand, an optimization model is established. Producing the similar type of products at the uniform cost may not attract the customer demand always. Instead of stabilizing the distribution of products according to the customer demand, it is an important one to design the model for enabling the products to produce at the variable cost of production, at the multiple price of sales and also at the altered distribution towards the demand for the main objective of optimizing the level of profit. On the other hand, the model is proposed with the assumption that the customer demand is not identical with the production costs, selling price and the supply of the items. In this study, limited production quantity and uniqueness of the products are considered. An optimization is achieved in producing, selling and distribution of costs, price and goods respectively. An illustration is presented for practicing in application on this model and graphical explanation is also provided to describe this model.

Keywords: Multiple production quantity- Variable cost of production, selling price and Distribution – Customer Demand – Optimization.

1. Introduction

Scarcity of matter is a pervasive aspect of human life and is the fundamental precondition of economic behavior. Expansion and development or improvements are the inevitable major aspect in supplying items to meet the demand of the customers. Supplying the goods to the consumers are always required modernization and some new techniques in product, price, promotion and physical distribution. This technique is more valuable to the producer to satisfy the consumers and to achieve the optimal level of sale the product at the profitable price. The availability of new concepts in production of products may design the marketing strategies. Selection of different types of aspects in marketing may be the reason for the achievement of objectives of the concern. The competitors views should also be considered to execute our strategy.

Most of the research on marketing is based on implementation of new concepts to the modern era. Because of boredom on uniqueness of the method of production and strategies, the consumers may not pay concentration to consume the products regularly. It affects the marketer to sustain the consumers for a long period. Hence, number of marketers are innovates new thoughts to attract the consumers with up to date technologies.

A good understanding of customer needs and wants, the competitive environment and the nature of the market represent the top required factors for the success of a new product. Cost, time and quality are the main variables that drive the customer needs. Aimed at these three variables, companies develop continuous practices and strategies to better satisfy the customer requirements and increase their market share by a regular development of new products. There are many uncertainties and challenges throughout the process which companies must face. The use of best practices and the elimination of barriers to communication are the main concerns for the management of NPD process. Commodity theory deals with psychological effects of scarcity. The theory also stress that any commodity will be important if it is unavailable.

According to the theory, variable production cost and selling price with variable distribution channel lead to optimal profits. The feeling of uniqueness may vary for different situations and persons; as such, it may be related to forces in a given situation that promote an extreme sense of high similarity, and dispositional factors that influence the high need for uniqueness across a variety of situations. Sirgy addressed the importance of scarcity in marketing strategy. Salespersons should apply new strategy on selling products or services. It will boost up the motivation of the targeted customers to handle the promotional information.



There are two strategies for variable production cost and selling price. The first one is concentrating on incurring the costs of production. With the low estimation of the cost the product is planned to produce. Then only the product can be sold at variable price of sales. Then the second one is concentrating on the quality and quantity of the production for sales. If the product is required to produce with high quality, it needs to spent more amount of money to the standard expected. On the other hand, if we want to sale at lower selling price, the production cost are determined low. As a result, multiple production cost and selling price can make a positive impact, but also may backfire if it is not launch properly. Therefore, if we combine the commodity theory and the need for the uniqueness theory, we can demonstrate that customers prefer the product which has more options in the cost level. At the same time the producer may also pick the profit from multiple price of sales of the product incurred variable cost.

Recently, many industries apply the strategy of limited production with variable cost and selling price. In inventory management, this problem is known as the “newsboy problem” or the “newsvendor problem”. The newsboy is a single period stochastic inventory problem which deals with stocking issues in today’s supply chainings. Weng analzed the coordinated quantity decisions between the manufacturer and the buyer in a newsvendor model. Dominey and Hill explored the effectiveness of approximating a compound poisson distribution in a newsboy model. Wang and Webster used loss aversion to model manager’s decision-making behavior in the single-period newsvendor proble. Shi et al. extended the multi-product newsvendor problem by incorporating the retailer’s pricing decision considering supplier quantity discount. From our literature search, no researches have been done on the newsvendor problem to consider the limited production quantity issues with variable production costs and selling price for variable distribution to meet the customer demand.

In this study, the supplier has to consider the uncertainty in customer demand. Having a good manufacturing and marketing strategy of the limited-edition production, the marketer should consider the changes of production costs and selling price to fetch multiple profit margin. We present an algorithm to derive an optimal model for variable production cost and selling price that the expected profit is maximized.

The remaining of this study is classified into 3 sections. In the section 2, the notations and assumption are described to reveal the basic concepts and conditions to design this model. The numerical presentation with graphical explanation is placed in the section 3. The final conclusion and the different possible ways of this approach of this are presented in the section 4.

2. Model Description:

Notation:

- $E\pi$ - The expected profit for the supplier
- Q - The production quantity for the supplier
- Q^* - The optimal production quantity for the supplier considering production quantity.
- Q_w - The production quantity for the supplier without considering limited production quantity.
- Q_w^* - The optimal production quantity for the supplier without considering limited production quantity.
- p_i - The selling price per unit without considering limited production quantity.
 $i = 1, 2, \dots, n.$
- P_j - The upper bound of selling price per unit when the production quantity is limited, j
 $= 1, 2, \dots, n.$
- $p_k(Q)$ - The selling price per unit with considering limited production quantity which is a function of production quantity. $k = 1, 2, \dots, n.$
- C_i - The production price per unit ; $C_i < p_k(Q)$, $i = 1, 2, \dots, n$
- S - The salvage value per unit $S < C_i$
- C_2 - The shortage cost per unit ; represents costs of best goodwill.
- x - The random demand with probability density function, $f(x)$ and cumulative distribution function, $F(x)$.

Assumptions:

Multiple production of the limited product is assumed. The supplier manufactures a batch of the products Q_i , and sells to the retailer (or) directly to the customers. The unit production price of the product is C_i . The unit production cost is assumed to be variable. The unit selling price is $p_i(Q_i)$. When the sale quantity is less than the production quantity Q_i , the left over is sold with a unit salvage value ‘S’. When the demand is more than the batch Q_i , shortage occurs. All shortage will be lost sale and the unit lost sale shortage cost is r . For the selling price $p_i(Q_i) = p_i$, the supplier will manufacture an optimal batch of Q_w^* . This is identical to the newsboy problem.

The suppliers expected profit function $E\pi$ is ;



$$E \pi(Q_i(w)) = \int_0^{Q_1(w)} \{(p_1 - c_1)x - (c_1 - s)(Q_1(w) - x)\}f(x)dx + \int_{Q_1(w)}^{\infty} \{(p_1 - c_1)Q_1(w) - x - Q_1(w)r\}f(x)dx + \int_0^{Q_2(w)} \{(p_2 - c_2)x - (c_2 - s)(Q_2(w) - x)\}f(x)dx + \int_{Q_2(w)}^{\infty} \{(p_2 - c_2)Q_2(w) - x - Q_2(w)r\}f(x)dx + \int_0^{Q_3(w)} \{(p_3 - c_3)x - (c_3 - s)(Q_3(w) - x)\}f(x)dx + \int_{Q_3(w)}^{\infty} \{(p_3 - c_3)Q_3(w) - x - Q_3(w)r\}f(x)dx$$

The suppliers optimal production batch is $F(Q_i^*(w)) = \frac{(p_i - c_i + r)}{p_i - s + r}$ where $F(x)$ is the cumulative distribution function of x . If the supplier manages the limited production batch, then the customers perceived valued and purchase decisions are usually influenced by the law of scarcity[1]. The unit selling price $p_i(Q_i)$ of the limited quantity products is a decreasing function of Q_i .

However, the customer demand will decrease due to a higher selling price. i.e., the random demand of the products depends on production batch Q_i , because the higher production batch, will decrease the selling price, while the lower selling price will increase demand. i.e, the pdf $f(x)$, of the random demand x is a function of Q_i . The supplier expected profit function $E\pi$ is given as follows;

$$E \pi(Q_i) = \int_0^{Q_1} \{(p_1(Q_1) - c_1)x - (c_1 - s)(Q_1 - x)\}f(x)dx + \int_{Q_1}^{\infty} \{(p_1(Q_1) - c_1)Q_1 - x - Q_1r\}f(x)dx + \int_0^{Q_2} \{(p_2(Q_2) - c_2)x - (c_2 - s)(Q_2 - x)\}f(x)dx + \int_{Q_2}^{\infty} \{(p_2(Q_2) - c_2)Q_2 - x - Q_2r\}f(x)dx + \int_0^{Q_3} \{(p_3(Q_3) - c_3)x - (c_3 - s)(Q_3 - x)\}f(x)dx + \int_{Q_3}^{\infty} \{(p_3(Q_3) - c_3)Q_3 - x - Q_3r\}f(x)dx$$

Our problem can be formulated as , max; $E\pi(Q_i)$ -----(4)

3. An illustrative model:

In this section , a practical selling price and probability distribution are applied to explain the results of the previous sections. Since the selling price is always influenced by the limited production quantity, the selling price per unit $p_i(Q_i)$ can therefore, we assumed as;

$$p_i(Q_i) = \frac{p_j - p_i}{\sqrt{Q_i}} + p_i \dots \dots \dots (5), \quad p_j > p_i > 0, Q_i \geq 1, i = 1,2,3 \text{ and } j = 1,2,3..$$

which means $p_i < p_i(Q_i) < p_j$ and is decreasing function of Q_i . For the supplier , the random demand is assumed to be uniformly distributed over the range 0 and $B(Q_i)$, where

$B(Q_i) = \frac{bp_i}{p_i(Q_i)} \dots \dots \dots (6)$, is a function of Q_i with positive constant b (b is the upper bound of the selling quantity). This means that a higher selling price would decrease demand. Thus the pdf of the supplier's demand is

$$f(x) = \frac{1}{B(Q_i)} \dots \dots \dots (7).$$

General Case:

$$\text{In this case , } B(Q_i) = \frac{bp_i}{p_i(Q_i)}$$

$$E \pi'(Q_i) = \left[\int_0^{Q_1} \{(p_1'(Q_1)x - c_1 + s) \frac{1}{B(Q_1)} dx + \int_{Q_1}^{B(Q_1)} \{(p_1'(Q_1)Q_1 + p_1(Q_1) - c_1 + r) \frac{1}{B(Q_1)} dx + B'Q_1/BQ_1 \right. \\ \left. p_1(Q_1 - c_1)Q_1 - BQ_1 - Q_1r\} + \int_0^{Q_2} \{p_2'(Q_2)x - c_2 + s\} \frac{1}{B(Q_2)} dx + \int_{Q_2}^{B(Q_2)} \{p_2'(Q_2)Q_2 + p_2(Q_2) - c_2 + r\} \frac{1}{B(Q_2)} dx + B'Q_2/BQ_2 \right. \\ \left. p_2(Q_2 - c_2)Q_2 - BQ_2 - Q_2r\} + \int_0^{Q_3} \{p_3'(Q_3)x - c_3 + s\} \frac{1}{B(Q_3)} dx + \int_{Q_3}^{B(Q_3)} \{p_3'(Q_3)Q_3 + p_3(Q_3) - c_3 + r\} \frac{1}{B(Q_3)} dx + B'Q_3/BQ_3 \right. \\ \left. p_3(Q_3 - c_3)Q_3 - BQ_3 - Q_3r\} \right]$$

$$E\pi''(Q_i) = p_1''(Q_1) \frac{Q_1^2}{2B(Q_1)} + [p_1''(Q_1) + 2p_1'(Q_1)] \frac{B(Q_1) - Q_1}{B(Q_1)} - [(p_1(Q_1) - s + r)f(Q_1) + \{(p_1(Q_1) - c_1)Q_1 - (B(Q_1) - Q_1)r\}[B'(Q_1)]^2 f'(B(Q_1)) + \{2B'(Q_1)[p_1'(Q_1)Q_1 + p_1(Q_1) - c_1 + r]\} + B''(Q_1)[(p_1(Q_1)Q_1 - c_1Q_1 - rB(Q_1) + rQ_1)] - r[B'(Q_1)]^2 f(B(Q_1)) + p_2''(Q_2) \frac{Q_2^2}{2B(Q_2)} + [p_2''(Q_2) + 2p_2'(Q_2)] \frac{B(Q_2) - Q_2}{B(Q_2)} - [(p_2(Q_2) - s + r)f(Q_2) + \{(p_2(Q_2) - c_2)Q_2 - (B(Q_2) - Q_2)r\}[B'(Q_2)]^2 f'(B(Q_2)) + \{2B'(Q_2)[p_2'(Q_2)Q_2 +$$



$$p_2(Q_2) - c_2 + r] + B''(Q_2) [(p_2(Q_2)Q_2 - c_2Q_2 - rB(Q_2) + rQ_2)] - r [B'(Q_2)]^2 \} f(B(Q_2)) + p_3''(Q_3) \frac{Q_3^2}{2B(Q_3)} + [p_3''(Q_3) + 2 p_3'(Q_3)] \frac{B(Q_3) - Q_3}{B(Q_3)} - [(p_3(Q_3) - s + r) f(Q_3) + \{ (p_3(Q_3) - c_3)Q_3 - (B(Q_3) - Q_3)r \} B'(Q_3)]^2 f'(B(Q_3)) + \{ 2B'(Q_3) [p_3'(Q_3)Q_3 + p_3(Q_3) - c_3 + r] + B''(Q_3) [(p_3(Q_3)Q_3 - c_3Q_3 - rB(Q_3) + rQ_3)] - r [B'(Q_3)]^2 \} f(B(Q_3)). \dots\dots\dots(9).$$

From (9), it is hard to prove the concavity of $E\pi(Q_i)$. The numerical example is provided to illustrate the model.

Example: $P_1 = 250, p_1 = 120, P_2 = 240, p_2 = 110, c_1 = 100, c_2 = 120, c_3 = 110, P_3 = 260, p_3 = 130, b = 1500, s = 50, \text{ and } r = 10.$

$$E\pi(Q_i) = E\pi(Q) = (368675000Q^{\frac{1}{2}} - 15430523Q - 3538253Q^{\frac{3}{2}} + 9350Q^2 + 2869Q^{\frac{5}{2}}) / (7800[13+12Q^{\frac{1}{2}}]) + (348326000Q^{\frac{1}{2}} - 14320692Q - 3226245Q^{\frac{3}{2}} + 8670Q^2 + 2569Q^{\frac{5}{2}}) / (7150[13+11Q^{\frac{1}{2}}]) + (369217000Q^{\frac{1}{2}} - 15442654Q - 3539266Q^{\frac{3}{2}} + 9380Q^2 + 2893Q^{\frac{5}{2}}) / (8450[13+13Q^{\frac{1}{2}}])$$

$$E\pi''(Q_i) = E\pi''(Q) = (13274300000Q^{\frac{1}{2}} - 43163916000Q - 1423124836Q^{\frac{3}{2}} - 516728148Q^2 - 4736760Q^{\frac{5}{2}} - 3537257Q^3 - 962064Q^{\frac{7}{2}}) / (7800Q^2 [13 + 12Q^{\frac{1}{2}}]^3) + (12539736000Q^{\frac{1}{2}} - 3928364000Q - 1217234664Q^{\frac{3}{2}} - 483637148Q^2 - 4238660Q^{\frac{5}{2}} - 3233267Q^3 - 931864Q^{\frac{7}{2}}) / (7150Q^2 [13 + 11Q^{\frac{1}{2}}]^3) + (13291812000Q^{\frac{1}{2}} - 4319426000Q - 1426344636Q^{\frac{3}{2}} - 516937258Q^2 - 4739560Q^{\frac{5}{2}} - 35392570Q^3 - 973081Q^{\frac{7}{2}}) / (8450Q^2 [13 + 13Q^{\frac{1}{2}}]^3)$$

Since the above equation of $E\pi''(Q)$, the first and third term, second and fourth term, fourth and sixth term and fifth and seventh term are negative, since $Q > 109$. This means that $E\pi''(Q) < 0$. Therefore, $E\pi(Q)$ is concave. The root of $E\pi'(Q) = 0$ and it is located in the interval [500 , 1000 , setting $E\pi'(Q) = 0$ which implies $Q^* = 795.70$. The selling price per unit is $P(Q^*) = \$ 186.70$ and $E\pi(Q^*) = \$ 10246$, $Q_w^* = 736$, $E\pi(Q_w^*) = \$ 8076$.

Therefore, the percentage profit increase is $\frac{E\pi(Q^*)}{E\pi(Q_w^*)} - 1 = 26.9\%$.

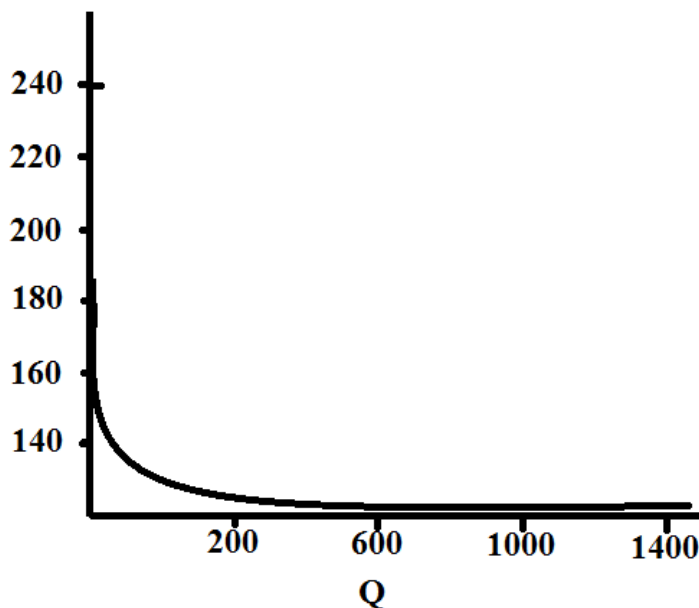


Fig. 1. The shape of $p(Q)$

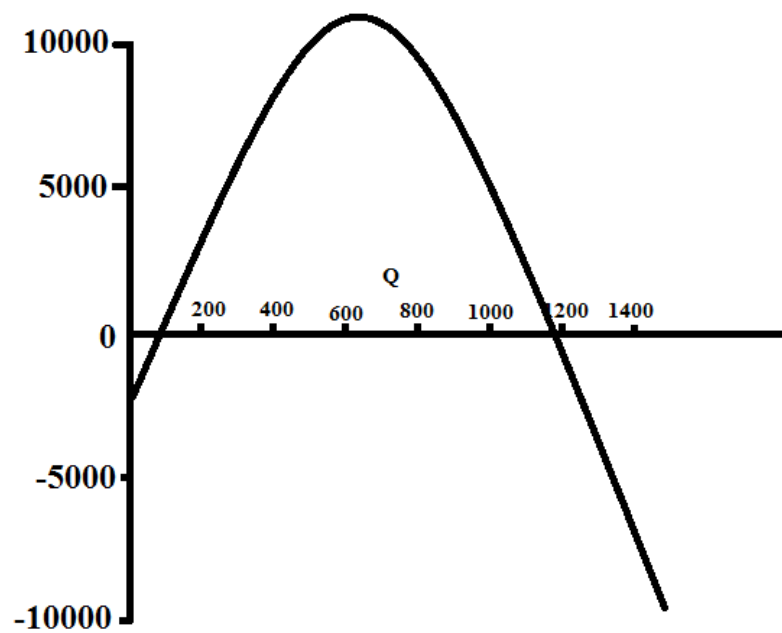


Fig.2 . The shape of $E\pi(Q)$

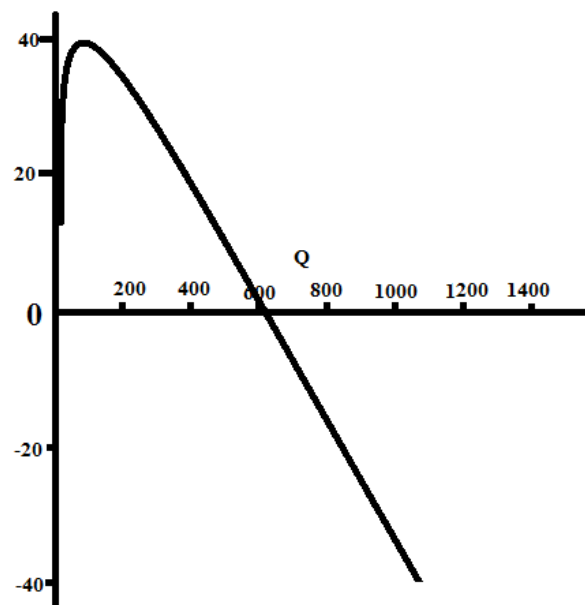


Fig 3. The shape of $E\pi'(Q)$

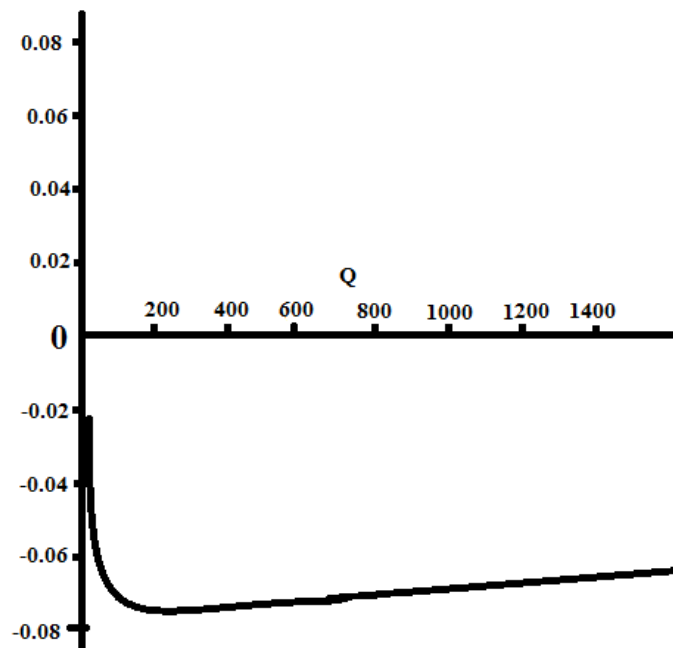


Fig.4. The shape of $E\pi'(Q)$

4. Conclusion

In this study, we derive a newsvendor problem model for optimizing the profit and sales by varying the production cost and selling price with different distribution level to meet the customer demand. The concept of production cost at the multiple level may help the vendor to satisfy the demand of various level s of the customers. In this method, the production costs can be properly maintained and controlled up to the level of attaining optimal profit by selling the product produced at variable costs. And also, the selling price can be fixed according to the level of expectation of the concern which understand the new technology to attract the consumer demand. This enables the firm to increases the product selling prices due to exclusive distribution outlets. In analyzing the system, we provide managerial insights to decision makers in planning production quantity and selling in order to derive the optimal profit. Controlling the unwanted costs and maintain the quality to supply the product at the various selling price are the key concept of this model.

Illustrative case studies, numerical examples and graphical representation are presented to demonstrate the proposed model. The numerical example show that the percentage profit increase is fairly significant. Most of the past researches focused on launch timing and reciprocal effects. The variable production costs, selling price and distribution are highly influenced for the optimal profit . For simplicity, the unit production cost is assumed to be fixed.

Future researches are suggested to consider an optimization model for products with unlimited quantity. While designing a model for unlimited quantity, the marketing strategy can be set up for a period with adequate stock and the additional costs incurring for holding the stock till it is sold, may also be considered. Whatever the concepts and technology followed in planning the model for optimization of profit, the researcher must concentrate on the customer demand the key element for the maximization of sales and profit.

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