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# Fuzzy Real Options Analysis Applied to Urban Renewable Energy Investments

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**Abstract:** The application of real options and fuzzy real options to renewable energy investment decisions is explored in the context of the valuation of three urban rooftop solar projects. Four real options methods were used to analyse the abandonment option present within these projects. Two of these methods, Fuzzy Black-Scholes and Fuzzy Binomial, used fuzzy numbers for the cashflow and salvage inputs. The resulting European put valuations for the option to sell the projects off for salvagewere consistent across the various techniques, including both classical and fuzzy. The real options present within the solar projects consistently added value to the adjusted net present values of the projects, which should improve their investment prospects. Additionally, we discuss the role of using fuzzy options pricing techniques as opposed to traditional real options and their usefulness to the practitioner.

## 1. Context[1]

In response to rising pressure from growing urban populations and their demands on infrastructure, significant investment is needed forthe development of urban energy systems(Townsend 2013).However, innovative renewable energy and energy efficiency (referred to hereafter as "smart energy") projects still have difficulty in raising capital(Merk et al. 2012; OECD 2015). These types of projects are susceptible to policy risks which can make potential investors reluctant to invest in energy infrastructure (Foxon et al. 2005; Mitchell et al. 2006). One key approach towards encouraging investment in smart energy projects is appropriate project valuation, especially where there is risk and flexibility present.

Smart energy projects are traditionally assessed by using the Net Present Value (NPV) technique, which is based on Discounted Cash Flow (DCF) analysis (Brealey et al. 2011). However, this type of analysis has a tendency to undervalue projects that require long timeframes or which have managerial flexibility in their executions (Amram and Kulatilaka 1998), such as smart energy infrastructure projects(OFGEM 2012). As stated by Fernandes et al. (2011), "Traditional evaluation models relying mainly on discounted cash-flows fail to assess the strategic dimension of [renewable energy] investments and do not allow for properly dealing with the risk and uncertainty of these particular projects." NPV has the disadvantage that it requires "the assumption of perfect certainty of project cash flows." (Miller & Park 2002), and the use of NPV has important consequences when valuing long term projects, which would include many energy infrastructure projects, "such that the far future may appear worthless" (Aspinall et al. 2015). Copeland and Antikarov (2003) assert that NPV "systematically undervalues every project" due to the fact that "it fails to capture the value of flexibility." As a result, some even go so far as to declare the use of NPV in environmental decision making as invalid and unethical (Robinson 1996).

In reality, most projects have many options available, such as the option to delay the start of the project, the option to expand, and the option to abandon. In order to address the shortcomings of traditional project valuation, Real Options Valuation (ROV) allows the elements of project flexibility and uncertainty to be factored into the project valuation by modelling them after financial options, namely calls (options to expand) and puts (options to contract) (Trigeorgis 1996). Therefore, well-established financial option pricing techniques, such as Black-Scholes(Black & Scholes 1973) or Cox Ross Rubinstein (CRR) Binomial Trees(Cox et al. 1979), can be applied to determine the value of project flexibility, which can then be used to enhance the expected value of the project.

[1] Abbreviations:
ROV = Real Options Valuation
FROV = Fuzzy Real Options Valuation
NPV = Net Present Value
DCF = Discounted Cash Flow
FIT = Feed-in-Tariff

However, while ROV captures uncertainty in project cashflows and options, the financial options valuation techniques still have the drawback of requiring quite precise input parameters, and real options values can be sensitive to small changes in the underlying inputs.(Dixit & Pindyck 1994)In order to overcome these limitations, the use of fuzzy options valuation techniques has been proposed as a way to value real options when inputs like cashflow forecasts are less precise (Wang & Hwang 2007).In fuzzy real option analysis, instead ofthe inputs having one specific value, they are formed of a range of values in order to model the natural uncertainties that may arise in cashflow or other parameters. The motivation behind fuzzy real options is ultimately to provide the user with a practical means of handling project uncertainty and forecasting future cash flows arising from the value of such options.As stated by Collan et al. (2009), "This means that fuzzy sets can be used to formalize inaccuracy that exists in human decision making." In the domain of smart energy infrastructure,which is subject to various uncertainties such as price and policy risk, this is characteristic is important, because when it comes to investing in renewable energy projects, "getting the numbers right […] is far from simple" (Abadie and Chamorro 2014).

## 2. Objectives

Against this background, the main aim of this work is to explore the application of fuzzy real options analysis to urban renewable energy infrastructure. In so doing, a the use of real options models will be explored, focusing in particular on their use of fuzzy numbers to capture the value of real options under the presence of uncertainty. The classical (non-fuzzy) Black-Scholes and CRR models will be compared with the fuzzy Black-Scholes and fuzzy CRR models in order to determine whether real options can be used to improve smart energy investment valuation, and also to evaluate their consistency and ease of use.

We will apply Real Options Valuation (ROV) and Fuzzy Real Options Valuation (FROV) to three rooftop solar projects in London in order to determine whether the additionality of project flexibility adds to the overall value of the projects. To this end, three projects(Brixton Energy 2012a; Brixton Energy 2012b; Brixton Energy 2013) had their proposed initial investments and potential cashflowsanalysed using Discounted Cash Flow(DCF) in order to establish their nominal NPV. These inputs were used to feed into the ROV and FROV pricing models so that the resulting option prices could be compared for consistency, and in order to investigate whether adding additional flexibility to the inputs in the form of "fuzziness" returned results consistent with non-fuzzy ROV models.

The analysis continues in Section 3 with a literature review and survey of the relevant theoretical and practical works underpinning the models used here. Section 4 sets out a detailed description of our case study of rooftop solar projects, and the methodology for our real options analysis of these case studies is discussed in Section 5. The results of the solar projects analysis are presented in Section 6, followed by a critique of the usability of fuzzy real options in Section 7. Section 8 presents our final conclusions.

## 3. Literature Review

Fernandes et al. (2011)presents a review the application of real options analysis to investments in non-renewable and renewable energy sources thus demonstrating the positive impacts of ROV on assessing these types of projects.We focus on real options as applied to renewable energy (RE) investments, because the case study that this work focuses on is based on solar power generation, however most existing literature on the application to RE investments applies to wind and hydropower.

Zeng et al. (2015) explores the application of ROV to solar projects that generate part of their income from renewable energy credits (REC), which are subject to their own price uncertainty. This paper implements a Monte Carlo simulation and optimization method based on approximate dynamic programming to solve their real options model to determine the optimal time for buyback of third-party owned generating assets. This paper does not focus on overall project valuation, but rather on decision timing, and the only source of uncertainty in the model are REC prices.

Venetsanos et al. (2002) applies real options valuation to renewable energy generation in the form of a wind farm. This approach used the Black-Scholes options pricing model, and found that for their wind farm business case, the ROV-enhanced NPV was greater than the traditional NPV.Boomsma et al. (2012) use real options to investigate the effects of different renewable energy support schemes, such as feed-in-tariffs and renewable certificate trading, on wind farm investments. This paper compared different RE support schemes, and accounts for uncertainty in capital costs, electricity prices, and subsidies. They used a least-squares Monte Carlo options pricing model and used the option values to determine optimal time to investment and also optimal RE support schemes, finding that feed-in-tariffs encourage earlier investment. Similarly, Abadie and Chamorro (2014)explore the valuation of the option to delay an irreversible investment in wind energy in a decentralised, deregulated energy market setting under various RE support schemes. Their approach uses mean

reverting trinomial and binomial lattice options pricing models and considers uncertainties in electricity prices, wind load, and renewable credit prices. They found that initial one-off subsidies have a stronger effect than ongoing mechanisms like feed-in-tariffs.Finally, Min et al. (2011) performed an analytic analysis of optimal entry and exit times for renewable generation assets by considering operations and maintenance (O&M) costs as the primary source of cost uncertainty. Their work assumes that the O&M costs follow a geometric Brownian motion (GBM) process, and models the problem as an optimal stopping problem, or an abandonment option.

Monjas-Barroso and Balibrea-Iniesta (2013)also investigate wind projects with real options present from policy frameworks in three European countries using both Monte Carlo and binomial tree techniques to value the call options, and they found similar results from both techniques as long as the number of nodes and iterations are sufficiently high. They compared RE investment frameworks in Finland, Denmark, and Portugal, and found that Finland had the strongest economic support for wind energy. Their primary sources of uncertainty were construction costs and electricity prices.Thomas and Chrysanthou (2011) apply Black-Scholes real options analysis to find the optimal time for investment in nuclear power, offshore wind, and onshore wind, and found that nuclear power is most economically viable under contemporary market and policy conditions. The main source of uncertainty was electricity prices,although the effect of renewable obligation credits was also explored.

Two papers consider the use of real options analysis using binomial pricing techniques as applied to renewable energy policy in Taiwan (Cheng et al. 2011; Lee and Shih 2010). Cheng et al. (2011) applies a modified binomial model based on sequential compound options to explore theeffects of uncertain future electricity demand on lead time for implementing a clean energy policy. Their main forms of uncertainty were energy demand, for which GDP forecasting was used as a proxy. They use a multistage binomial tree to value the real options and therefore attempt to optimise policy strategy in Taiwan.Lee & Shih (2010) also use binomial options pricing to explore the interactions of RE policies and the uptake of wind energy generation. They found that feed-in-tariffs for wind energy in the context of Taiwan negatively impacted policy return on investment, and their models used the cost of non-renewable energy, the cost of renewable energy, and levels of policy support as their inputs.

As can be seen, all of these papers use various real options pricing approaches for the valuation of investment in renewable energy,however none of them use fuzzy real options analysis, which is considered in the works below.

The application of fuzzy numbers to finance started with Buckley's paper (1987) on fuzzy set theory as applied to cashflow analysis and the ranking of fuzzy investment alternatives. Fuzzy numbers have since been used in real option valuation, as presented in several papers. In particular,Carlsson & Fullér (2003)and Collan et al. (2003) both describe a model of pricing real options using a Fuzzy Black-Scholes (FBS) model based on fuzzy trapezoidal numbers. In these papers, they apply the FBS pricing approach to real options present in giga-investments with lifetimes of 15-25 years.Carlsson & Fullér (2003)concluded that a fuzzy real options model "that incorporates subjective judgments and statistical uncertainties may give investors a better understanding of the problem when making investment decisions."Following on from this work, the fuzzy payoff method, which is based on the Datar-Mathews payoff method, is explored in Collan et al. (2009) and Collan et al. (2012). The latter work incorporates the use of a credibility factor to weight the fuzzy inputs according to confidence.

There have been several works that also look at fuzzy implementations of CRR binomial tree options pricing. Ho and Liao (2010; 2011) present a fuzzy Cox-Ross-Rubinstein (FCRR) binomial tree model based on fuzzy triangle numbers in their papers and propose a method for computing the mean value of a fuzzy number so that it can be compared with a crisp number. Similar implementations of fuzzy binomial tree models are also discussed by Muzzioli & Torricelli (2004) and Yu et al. (2011).Muzzioli & Torricelli (2004) describe a FCRR approach to financial options pricing using a triangle fuzzy volatility input. This paper shows some comparisons of FCRR options prices compared with standard CRR. Yu et al. (2011) also describes a FCRR model using a fuzzy triangular volatility parameter, however they do not perform a numerical implementation or compare the results of their FCRR implementation with the standard CRR.None of these papers uses a certainty parameter. None of these works are specific to the use of fuzzy real options analysis in deployment of smart energy infrastructure, nor do they discuss application to specific case studies, which this paper will address.

Our work differs from these previous papers in several ways. The first set of these papers focuses on real options as applied to renewable energy projects, however they do not explore the residual value of solar assets, which is a key point of managerial flexibility with response to policy change risks. Additionally, these papers use traditional real options valuation rather than fuzzy real options valuationand most of them pertain to optimal investment time rather than overall project valuations. The second set of papers focuses on various fuzzy real options valuation models, however they do not apply these techniques to the valuation of renewable energy projects. Furthermore, very few of these papers provide a direct comparison of fuzzy real options valuation compared to classical real options valuation, especially with respect to increasing fuzziness. This

paper distinguishes itself from the previous literature in that it applies multiple fuzzy and classical real options valuation techniques to solar powered renewable energy projects to check their consistency of performance and ease of implementation, and furthermore performs a sensitivity analysis to investigate the behaviour of fuzzy real options valuation techniques under the influence ofan increasing uncertainty (fuzziness)parameter and proposes an altered FCRR model that provides more stable fuzzy options valuation.

## 4. Case Studies

Three projects(Brixton Energy 2012a; Brixton Energy 2012b; Brixton Energy 2013) had their proposed initial investments and potential cashflowsanalysed using Discounted Cash Flow(DCF) in order to establish their nominal NPV. This was done in order to establish a baseline project valuation before the application of real options analysis, and to determine the underlying value of the asset along with the volatilities of the cashflows, which are inputs required by options pricing models. The inputs for each business model were pulled from the three share offering documents (Brixton Energy 2012a; Brixton Energy 2012b; Brixton Energy 2013), which describe three rooftop solar projects in London, along with their cashflows. According to these share offering documents,the revenues for these projects were Feed-in-Tariff (FIT) contracts with lengths of 20 years (or 25 years in the case of Brixton 1) were agreed, whereby each project would receive a fixed payment for each kilowatt of electricity generated. The outgoings included maintenance and insurance costs. The Feed-in-Tariffs would provide the revenues that would nominally pay back the initial investment costs, service the annual running costs, and give a return on investment to the shareholders. Because the FIT agreements are fixed, the main uncertainties in the business cases were the estimates for the running costs.

| PROJECT | Size | Investment | IRR | NPV | NPV w/ Extension | Salvage |
|---|---|---|---|---|---|---|
| Brixton 1: Capex | 37 kW | £75,000 | 6% | £5,696 | £10,046 | £13,263.98 |
| Brixton 2: Capex | 45 kW | £61,500 | 10% | 26,497 | 35,826 | 16,131.87 |
| Brixton 3: Capex | 52.5 kW | £67,000 | 8% | 17,668 | 23,192 | 18,820.51 |
| Brixton 1: Loan | 37 kW | £10,000+Loan | 4% | -4,595 | -245 | 13,263.98 |
| Brixton 2: Loan | 45 kW | £10,000+Loan | 11% | 18,309 | 2,307 | 16,131.87 |
| Brixton 3: Loan | 52.5 kW | £10,000+Loan | 5% | -2,511 | 3,013 | 18,820.51 |

Table 1: Three solar projects are shown, financed two different ways, where “Capex” indicates an initial upfront investment, and “Loan” indicates loan financing. The expected net cashflow is shown for the first year of the project, along with the projected salvage value of the solar arrays at year 20.

The business cases also described an intended repayment scheme for local shareholders and charities, but in order to simplify our models, we have performed two types of discounted cashflow analysis (DCF) for each project: one as if all of the initial investment capitalhas been funded from internal budgets; and the other on the basis that the investment capital comes from a ten year business loan. We therefore model the cashflows for the three projects were modelled, first, as if they are outright capital expenditure (capex) investments, and second, as if a loan of 10% interest on a down payment of £10,000 is used to finance the projects. Both cash flow analyses are conducted under the assumption that there are no shareholders or dividends to pay. Our result is six DCF analyses, yielding six Net Present Value (NPV) project valuations, two for each project, respective to the means of financing, as shown in

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Table **1**. These initial valuations serve as the original figures against which our ROV enhanced valuations will be compared.

For real option analysis to be relevant and effective, the project must have some element of managerial flexibility(Trigeorgis 1996; Dixit & Pindyck 1994; Amram & Kulatilaka 2000). The public share offering documents(Brixton Energy 2012a; Brixton Energy 2012b; Brixton Energy 2013)gave no speculation as to what would happen to the projects after the expiration of the FIT contracts, despite the fact that the lifetime of photovoltaic (PV) cells could be up to 20 years longer than the duration of those contracts(Jordan & Kurtz 2013). However, due to policy and market uncertainty, there are several scenarios that would prevent the continuing operation of the solar installations: the rooftop lease might not be renewed, or the purchase price for the electricity may be too low to generate sufficient revenue. If the projects need to be discontinued after the FIT expires, the PV arrays could be sold for salvage since they would still have operational lifetime remaining. Here lies the source of the optionality present in this project: the managers will be able to execute some oversight in reaction to the prevailing conditions and new information, through which they can attempt maximise future revenues.

The ability to sell the array for salvage can be modelled as an abandonment option because it would be exiting the market and shutting down the project. Since it occurs after a fixed term, this scenario can be priced as if it were a European put option since that is the method for pricing options to sell an asset to the market at a fixed time(Brealey & Myers 2010). In short, the salvage options give valuations for the opportunity – but not the obligation – to sell the photovoltaic arrays for salvage at the end of the FIT contracts. For this reason we include the value of these salvage options in our analysis of the project valuations.

In order to perform a real options analysis on these three projects, a number of values are needed for each project, namely: initial investment, NPV, discount rate, revenues, outgoing expenses, variation in cashflows (volatility), and salvage value for each array. The initial investments and NPVs are obtained as described above. The discount rate was established to be 5% since the cashflows are fixed in advance and therefore fairly low risk.

The revenues for each project are basedon the amount of electricity projected to be generated each year, with a FIT payment per kWh produced. Solar PV cells do degrade in efficiency over time, and the public share offering documents have factored in a decrease in productivity of 1% per year, although this is higher than observed degradation in modern solar cells (Jordan & Kurtz 2013). However, the FIT agreement is fixed to inflation, so payments will also be adjusted according to RPI every year. Inflation in RPI is generally upwards, and to produce a forecast of inflation rates, a Monte Carlo analysis was performed on inflation data from 1986 to the present, which gave a projected rate of inflation of 3% in 20 years. The payments for generation were adjusted upwards annually according to this level of inflation, as were the variable outgoing maintenance and running costs. Because of the fixed nature of the FIT contracts, the volatility of energy prices does not affect the project revenues directly. The underlying assets for these analyses are the present value of the total cash flows for each of the solar projects and the intrinsic value of the photovoltaic arrays.

The salvage value for each array iscalculated by assuming that it would be possible to receive 50 pence per kilowatt of generating PV, and then discounting that total value back to the present value. This was done according to information given by McCabe (2010), which stated that $1 per kilowatt is not uncommon. The same paper states that banks often calculate salvage value for PV arrays by estimating them at 15-25% of the initial investment. However, as can be seen by comparing the project costs of Brixton 1 (£75,000 for 37kW) with the later project Brixton 3 (£67,000 for 52.5kw), PV and installation costs have dropped dramatically in the past few years, so it is more consistent to fix the salvage price to generating capacity of the arrays rather than to initial investment amounts.

The future expected cashflows are calculated using the following assumptions. The first assumption is that even if the FIT can be renewed in the future, it will be at a lower rate on the basis of recent policy changes with respect to FIT tariffs from the UK Department of Energy & Climate Change (2015). According to the current policy changes, agreed FIT payments for solar schemes are 4.59 pence per kWh as of January 2016. It is very difficult to predict energy tariffs 20 years in the future, whether subsidised or not, but for the purposes of our calculations, we have assumed a FIT rate of 4 pence per kWh and 1 pence per kWh for export for any renewed FIT contract beyond year 20. We have also assumed two outcomes for the purposes of calculating future expected cashflows: 1) a 50% chance of renewing a contract with the updated FIT terms, or 2) a 50% chance of failing to secure a new FIT contract or rooftop lease. We also assume that the fixed annual costs remain constant over the lifetime of the project.

With these parameters in place, we then calculated the expected average cashflows for each project from the years beyond the standard FIT contract, as follows. We summed the present value of the cashflows from year 20 (or 25 for Brixton 1) to year 40. We averaged this sum with zero to reflect the 50% chance of receiving no revenues over those years. The standard deviation was also calculated, and these were used to

calculate the variance in the projects' returns at the time of the expiry of the option (either year 20 or 25). The coefficient of variation (CV) was then calculated as the standard deviation divided by the average expected cashflows. This way, the variance, $\sigma^2$, of the projects' returns was calculated using

$$\sigma^2 = \frac{\ln(\mathrm{CV}^2 + 1)}{t} \quad (1)$$

where CV is the coefficient of variation, and $t$ is the time until the option expires (20 or 25 years, respectively). The square root of the variance is the volatility, $\sigma$, and according to this analysis, the volatilities of all three of the projects were consistently 21%. It is also notable that the expected cashflows (asset value) for each project were the same regardless of financing due to the fact that a ten-year loan would be paid off before the timeframe being considered, rendering the cashflows from year 20 (or 25) onward equivalent. This meant that only one option price per project needed to be calculated, irrespective of whether it was cash or loan financed. If the loan repayment had stretched into the option time horizon being considered, this would not have been the case.

Now that the inputs necessary to price an option (asset price, strike price, lifetime, and volatility) are established, the salvage option price is calculated using the four option valuation techniques as described below.

## 5. Methodology

In order to determine whether fuzzy real options can be used to improve smart energy investment valuation, and to evaluate their consistency and ease of use, the traditional real options valuations techniques (Trigeorgis 1996; Amram & Kulatilaka 2000; Copeland & Antikarov 2003) must be compared with their respective Fuzzy Black-Scholes and Fuzzy CRR Binomial Tree methods. To this end, the Black-Scholes and CRR binomial tree models are utilised because they are the standard ROV models for use in continuous-time (analytic) and discrete-time scenarios, respectively (Martínez Ceseña et al. 2013). Because our business cases and their resulting cashflows consist of one option scenario that is fixed in time, both the Black-Scholes and the CRR binomial tree approaches are applicable. The two results from the two FROV models are compared with the results from the ROV modelsto explore whether additional "fuzziness" or uncertainty in the inputs yields options values that are consistent with the ROV models, and to determine if the ability to take fuzzy inputs makes the FROV models easier to use than the traditional ROV models.

The options prices for three rooftop solar projects were calculated using four valuation techniques: classical Black-Scholes(BS) (Black & Scholes 1973; Merton 1973); Fuzzy Black-Scholes (FBS) (Carlsson & Fullér 2003; Collan et al. 2003); traditional Cox, Ross, and Rubinstein (CRR) binomial tree(Cox et al. 1979); and Fuzzy CRR Binomial Tree(FCRR)(Liao & Ho 2010; Ho & Liao 2011; Yu et al. 2011; Muzzioli & Torricelli 2004).The Black-Scholes and CRR binomial tree techniques were used because these are the most common techniques used in existing real options calculators.In the case of the Black-Scholes, Fuzzy Black-Scholes (FBS), and CRR binomial tree analyses, we implement the models exactly as described in the previous literature so that our findings can be directly comparable to those found in the previous literature. In the case of the Fuzzy CRR (FCRR) model, we have implemented our model differently to the previous literature for two reasons: firstly, in order to draw comparison between the different models, we needed to ensure that they all took similar inputs, and secondly, we altered the FCRR pricing method in order to avoid a distortion in the option prices, which is explained in detail below. The options were modelled as European puts with a fixed lifetime of 20 years (or 25 for Brixton 1), which corresponds to the expiry of the Feed-in-Tariff (FIT) contract for each of these projects.

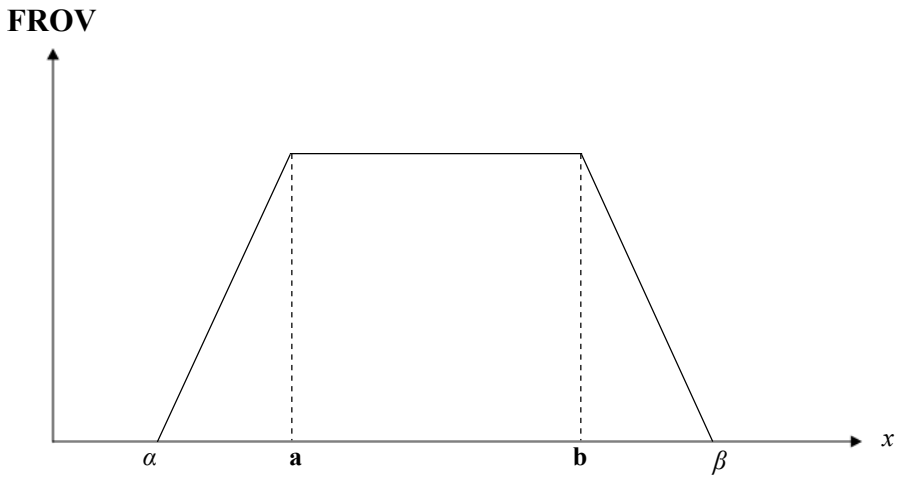


Figure 1: Fuzzy Trapezoidal Number (Collan et al. 2003)

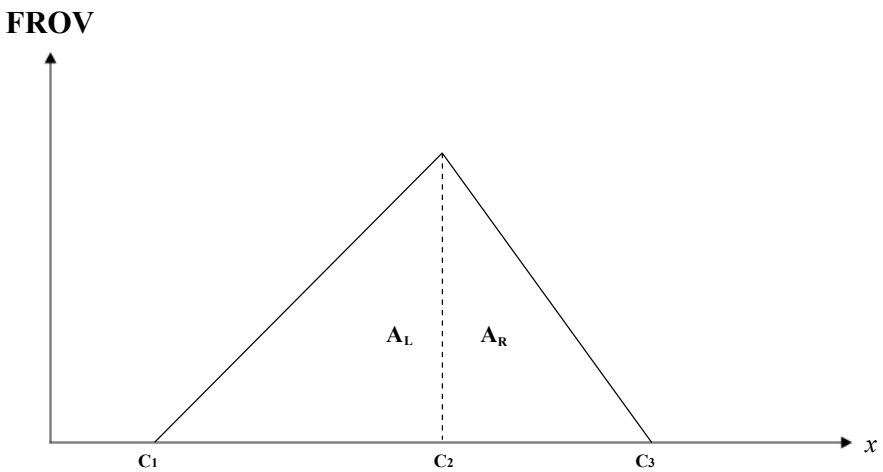


Figure 2: A Fuzzy Triangle Number (Ho & Liao 2011)

In order to compare the fuzzy options prices calculated from the FBS and FCRR techniques, the fuzzy outputs must be converted to a crisp number. As shown by Carlsson & Fullér (2003b), the possibilistic mean value of a fuzzy trapezoidal number *A'* can be calculated as

$$E(A') = \frac{a+b}{2} + \frac{\beta-\alpha}{6}, \quad (2)$$

where *a*, *b*, *α*, and *β* are as depicted in Figure 1 above, and *a* and *b* correspond to the lower and upper bound of the fuzzy trapezoidal number, and *α* and *β* correspond to the "fuzzy" parameter, or an additional range of possibility above or below the core [*a*,*b*] range. This technique for finding the fuzzy mean value essentially operates as a weighted average. Therefore if the trapezoidal fuzzy number is symmetric such that *α* and *β* are equal, then the second term in *E(A')* drops out and ( 2becomes a straightforward arithmetic average for a number *A'*=[*a*,*b*].

A similar technique is used for the triangular fuzzy numbers resulting from the FCRR model, in accordance with the approach described by Ho & Liao (2010; 2011). In order to derive the crisp option value from the fuzzy, a weighted arithmetic mean of a fuzzy number $V_n'$ is used as follows:

$$E(V_n') = \frac{(1-\lambda)c_1 + c_2 + \lambda c_3}{2} \quad (3)$$

where $c_1$, $c_2$, and $c_3$ are the range values of the fuzzy triangle number $V_n' = [c_1, c_2, c_3]$ (as shown in Figure 2), and $\lambda = \frac{A_R}{A_L + A_R}$. Because $A_L$ and $A_R$ are the areas of the triangles as defined by the values $c_1$, $c_2$, and $c_3$, $\lambda$ can be expressed as

$$\lambda = \frac{c_3 - c_2}{c_3 - c_1}.$$

Similar to the FBS model, if the triangle fuzzy number is symmetric such that $c_3 - c_2 = c_2 - c_1$, then the resulting crisp option price is equivalent to pricing a CRR option using $c_2$, the middle value.

We performed a sensitivity analysis to confirm that changes in the overall fuzziness of the inputs did not introduce a skew in the resulting fuzzy options pricing. To investigate the robustness of the fuzzy options methods with respect to increasing fuzziness, a series of call option prices were generated using the FCRR model (based on (Ho & Liao 2011; Yu et al. 2011; Muzzioli & Torricelli 2004; Liao & Ho 2010))where all of the inputs are kept static except the fuzziness parameter, which was increased in increments of 5%. The results of this procedure are displayed in Figure 3, which shows that the mean values of the fuzzy options prices that are produced increase exponentially with respect to the fuzzy parameter. This increase in mean option value comes from the positive skew in interim option values at each node in the CRR options lattice, and is due to fact that an increasingly fuzzy volatility leads to larger spreads in the "jump" factors that are used to construct the payoff lattice. In the CRR model, negative payoffs are discarded at each step in favour of zero, in accordance with the standard implementation of the payoff equations, which leads to an increase in option value proportional to the fuzziness. As stated by Liao & Ho (2010), "the characteristic of right-skewed distribution also appears in the FENPV of an investment project when the parameters (such as cash flows) are characterized with fuzzy numbers," so this appears to be an implementation decision, however as shown in Figure 3, this approach has the effect of overinflating option values when uncertainties increase. Given that FCRR options prices should remain stable and consistent with prices generated by other methods like FBS, our FCRR valuation method is implemented slightly differently.

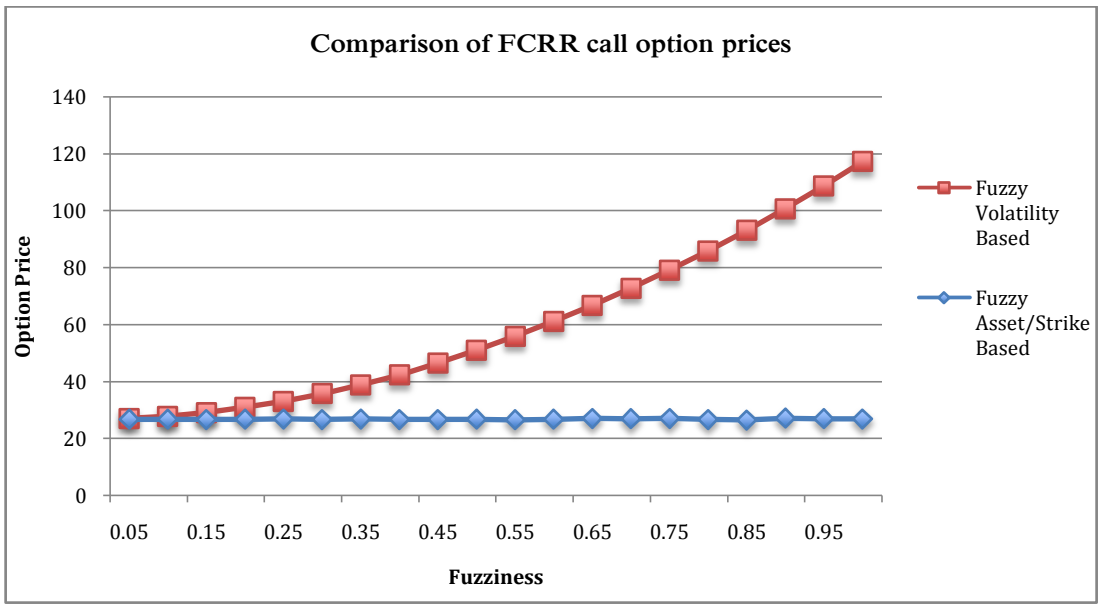


Figure 3: Call prices resulting from the Ho & Liao fuzzy volatility-based implementation of CRR compared with a fuzzy asset and strike price based implementation of CRR.

In order to decouple the skew in FCRR option values from the increase in fuzziness, we implemented the FCRR in a modified manner. In our FCRR model, instead of using the fuzzy parameter to generate a fuzzy volatility input, σ′, we instead kept the volatility constant while using the fuzzy parameter to generate a fuzzy triangle asset price $S_0'$ and a fuzzy triangle strike price $X'$. This means that the lattice jump factors are dependent on a crisp volatility value alone, preventing the introduction of an exponential rise in asset and option

values in the lattices, which in turn prevents the increase in mean option price. In this method, the up and down jump factors and their corresponding probability is calculated according to the traditional CRR method. The fuzzy asset and strike prices, $S_0'$ and $X'$, are used to create a fuzzy version of the payoffs for the option pricing lattice subject to the condition $\max\left(X' - S'_i u^i d^{n-i}, 0\right)$. By using this method of using fuzzy asset and strike price inputs, a fuzzy triangle European put option value is calculated, but without the skew introduced by using a fuzzy volatility input.

As compared with the Ho & Liao (2010; 2011) options prices shown in Figure 3, the mean call option prices that result from our model are stable with respect to fuzziness. As shown in Figure 4, the range of fuzzy call values increases as fuzziness increases, but the mean remains stable, as expected. This, in turn, yields option prices that are consistent with other pricing methodologies, rather than inflating the options valuation.

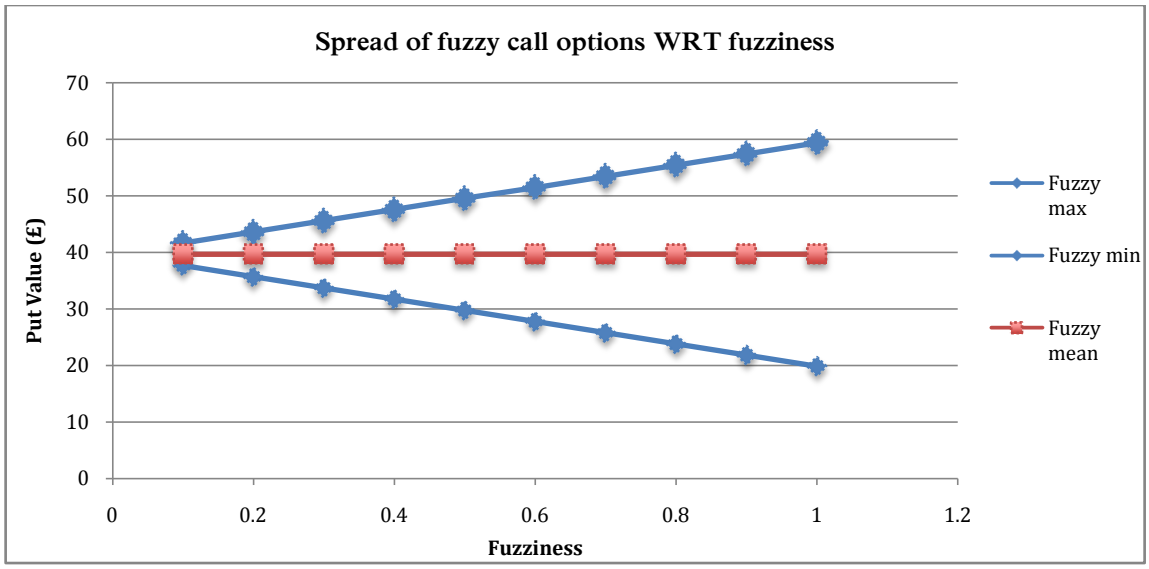


Figure 4: The spread of fuzzy option values increases with respect to increasing fuzziness, however the mean remains stable.

## 6. Results and discussion of the analysis

As discussed above, inputs that are symmetric around a crisp asset and/or strike value will yield mean option prices equal to their crisp counterparts (see Equations ( 2 and ( 3). However, as the symmetric case is also the most trivial case, it does not offer any insights into the sensitivity of the fuzzy options models when they are responding to human inputs that are less likely to be symmetric and more likely to incorporate a greater variance in estimates for the asset and strike inputs.Therefore a random element of fuzziness introduced into the strike and asset price inputs for the fuzzy models in order to replicate the vagaries of human uncertainty. In this way, we are able to explore whether these fuzzy real options models return values that were robust under slightly varying levels of fuzziness.

Using the inputs from the rooftop solar projects, the four options valuation models, both traditional and fuzzy, were checked for consistency. In order to do this, the put option valuation was calculated for the salvage option present after year 20 (25 in the case of Brixton 1) when the Feed-in-Tariff (FIT) contract expires, and yet the PV installations should still be generating useable electricity. In order to calculate the value of the salvage options, the average of the future expected cashflows was used as the underlying asset, and the salvage value of the PV arrays was used as the strike price. The technique that was used for calculating the salvage option is analogous to the approach of valuing an abandonment option (Brealey et al. 2011) due to the fact than an asset would be sold on after project abandonment. In this method, the contracted FIT cashflows for the first 20 (or 25) years are used to calculate the standard NPV, and then the uncertain cashflows for the years beyond to year 40 are used to calculate the expected cashflows.

We found that the existing fuzzy volatility-based FCRR models skewed the resulting option prices as the fuzziness parameter was increased (as shown in Figure 3). Thereafter, the model was modified such that the new fuzzy asset/strike-based FCRR model became stable with respect to fuzziness, or in other words, the mean option price neither increases nor decreases as the degree of fuzziness is changed. Instead, as expected, the spread of the resulting fuzzy option prices does increase, but so as along as this spread is symmetric, it should

have no effect on skewing the mean option price. We have therefore used this modified FCRR model in our real options analysis in order to yield stable results.

After the input parameters were established from the cash flow analysis of the three rooftop solar projects in Brixton, the data from each project was fed into the four options valuation models described previously: traditional Black-Scholes(BS); Fuzzy Black-Scholes (FBS); traditional Cox, Ross, and Rubinstein (CRR) binomial tree; and Fuzzy CRR Binomial Tree(FCRR). The NPVs and salvage values are shown for the projects in

| PROJECT | Size | Investment | IRR | NPV | NPV w/ Extension | Salvage |
|---|---|---|---|---|---|---|
| Brixton 1: Capex | 37 kW | £75,000 | 6% | £5,696 | £10,046 | £13,263.98 |
| Brixton 2: Capex | 45 kW | £61,500 | 10% | 26,497 | 35,826 | 16,131.87 |
| Brixton 3: Capex | 52.5 kW | £67,000 | 8% | 17,668 | 23,192 | 18,820.51 |
| Brixton 1: Loan | 37 kW | £10,000+Loan | 4% | -4,595 | -245 | 13,263.98 |
| Brixton 2: Loan | 45 kW | £10,000+Loan | 11% | 18,309 | 2,307 | 16,131.87 |
| Brixton 3: Loan | 52.5 kW | £10,000+Loan | 5% | -2,511 | 3,013 | 18,820.51 |

Table **1**. In this analysis, the fuzziness parameter was set to 5% in order to represent a reasonable level of uncertainty in the cash flows. All four options pricing models were implemented so that the resulting option prices could be compared for consistency, and in order to investigate whether adding additional flexibility to the inputs in the form of "fuzziness" returned results consistent with non-fuzzy options valuation models.

Because the fuzzy option value in the FBS model shiftsaccording to the changes in $\alpha$ and $\beta$, we used a Monte Carlo technique in order to get a truly average FBS option value to compare with the traditional Black-Scholes. The FBS model was run 100 times for each set of input parameters, with a small element of randomness introduced each time. The meanwas taken of the resulting option prices, which was then be compared with the traditional Black-Scholes price to see if the values were consistent and whether the FBS model was robust under the introduction of random variations in inputs.

A similar technique was applied to the triangle fuzzy asset and strike price inputs for the FCRR model. The fuzzy parameter was used to set the triangle points,$c_1$ and $c_3$, equidistant about the midpoint, $c_2$, and then multiplied them by a random number between 0 and 1 to deform the triangle distribution randomly around $c_2$. As in the FBS case, this has the effect of also skewing the mean option price, so in order to get a true average FCRR option value to compare against the traditional CRR case, the model was also run for 100 times for each set of inputs. Thecrisp average of these resulting options prices was then compared against the traditional CRR results.

The mean European put prices resulting from the four options pricing techniques were found to be

| PROJECT | BS Put | FBSput | ECRRput | EFCRRput | Avg | StdDev | Error |
|---|---|---|---|---|---|---|---|
| Brixton 1 | £2,046.70 | £2,045.45 | £2,019.11 | £2,020.83 | £2,033.02 | £15.10 | 0.74% |
| Brixton 2 | 2,439.64 | 2,442.19 | 2,393.75 | 2,394.53 | 2,417.53 | 27.03 | 1.12% |
| Brixton 3 | 4,327.33 | 4,329.62 | 4,284.98 | 4,290.35 | 4,308.07 | 23.68 | 0.55% |

consistent (see

Table **2**).Despite using fundamentally different techniques for calculating the put option prices, the resulting values were consistent with each other across each project within 2%. The consistency of the resulting option prices, regardless of the pricing method used, demonstrates that even when allowing for some fuzziness in the inputs, consistent option prices are obtainable.

| PROJECT | BS Put | FBSput | ECRRput | EFCRRput | Avg | StdDev | Error |
|---|---|---|---|---|---|---|---|
| Brixton 1 | £2,046.70 | £2,045.45 | £2,019.11 | £2,020.83 | £2,033.02 | £15.10 | 0.74% |
| Brixton 2 | 2,439.64 | 2,442.19 | 2,393.75 | 2,394.53 | 2,417.53 | 27.03 | 1.12% |
| Brixton 3 | 4,327.33 | 4,329.62 | 4,284.98 | 4,290.35 | 4,308.07 | 23.68 | 0.55% |

Table 2: The outputs of the European Put options pricing models for each project according to the following methods: Black Scholes (BS), Fuzzy Black Scholes (FBS), CRR binomial tree, and fuzzy CRR binomial tree (FCRR).

One of the main objectives of this work has been to verify the hypothesis of whether real options analysis increases the valuation of these smart energy projects in order to improve their investment potential. Since the put option prices generated were consistent across the four valuation techniques, the average put option price for each project was used to adjust the original net present value (NPV) of each project to produce an Enhanced NPV (ENPV). This was achieved by adding the value of the salvage options to the original NPV (Trigeorgis 1996; Amram & Kulatilaka 2000). By so doing, the ENPV will then take into account the salvage value and expected future cashflows beyond FIT expiry, which are the main inputs into the put option calculation, where the salvage value is the strike price, and the cashflows are the asset price.

| PROJECT | Investment | FIT NPV | 40 Year NPV | ENPV |
|---|---|---|---|---|
| Brixton 1: Capex | £75,000 | £5,695.67 | £10,045.95 | £7,728.69 |
| Brixton 2: Capex | £51,500 | 26,497.22 | £35,824.56 | £28,914.75 |
| Brixton 3: Capex | £67,000 | 17,668.42 | £23,192.40 | £21,976.49 |
| Brixton 1: Loan | £10,000 + Loan | -4,595.17 | -£245.00 | -£2,562.14 |
| Brixton 2: Loan | £10,000 + Loan | 18,309.45 | £27,637.00 | £20,726.98 |
| Brixton 3: Loan | £10,000 + Loan | -2,510.71 | £3,013.00 | £1,797.37 |

Table 3: The Enhanced Net Present Values (ENPV) resulting from combining the salvage option with the FIT NPV. As can be seen, the ENPV values lie between the FIT NPV and 40-Year NPV values, which is consistent with the fact that these options factor in a level of uncertainty in the expected cashflows.

The project valuations increased when real options analysis was used. Some of the original project NPVs were negative, indicating that their potential for success was low (Brixton 1: Loan and Brixton 3: Loan). The ROV-adjusted NPVs are all positive, showing that allowing for the optionality of selling the photovoltaic arrays for salvage does improve the investment potential for these solar projects. A summary of these results is given in Figure 5. These results are consistent with the behaviour of options prices themselves. The put prices can be interpreted as a forecast of project viability in that a higher put price indicates a higher likelihood of exercising that option.

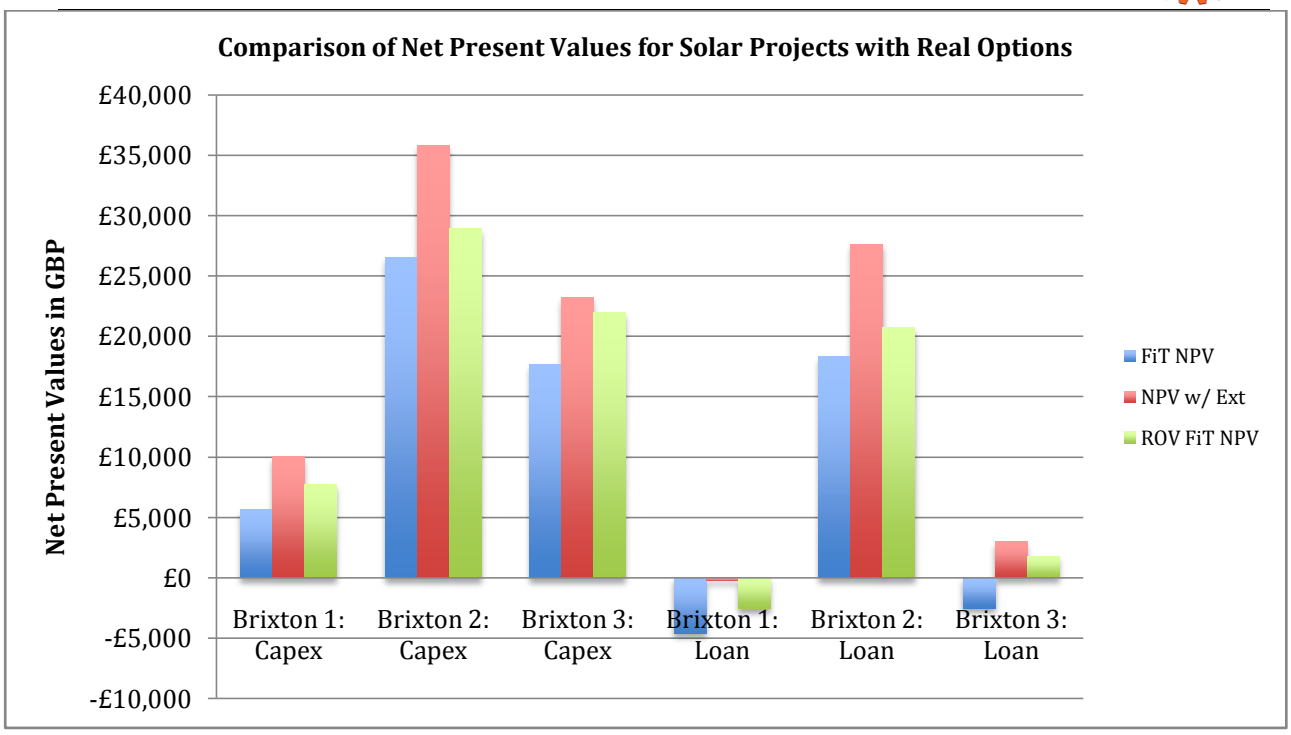


Figure 5:The original NPV of the three projects (with both up-front capital and loan financing cases shown) as compared with the adjusted NPV after the value of the abandonment option is added, along with the NPV of running the project with an extension to 40 years after FIT expiry.

To further investigate the use of option prices as an indicator of project viability, we calculated the internal rates of return for each of the six projects using their forecasted cashflows, and plotted these values against their respective ENPVs. Since IRR is related to NPV, and both are used as benchmarks of returns on investment, we would expect a linear relationship. The resulting graph (Figure 6) shows this linear relationship between the IRRs and ENPVs, with an $R^2$ value of 84%, albeit with a small sample size.

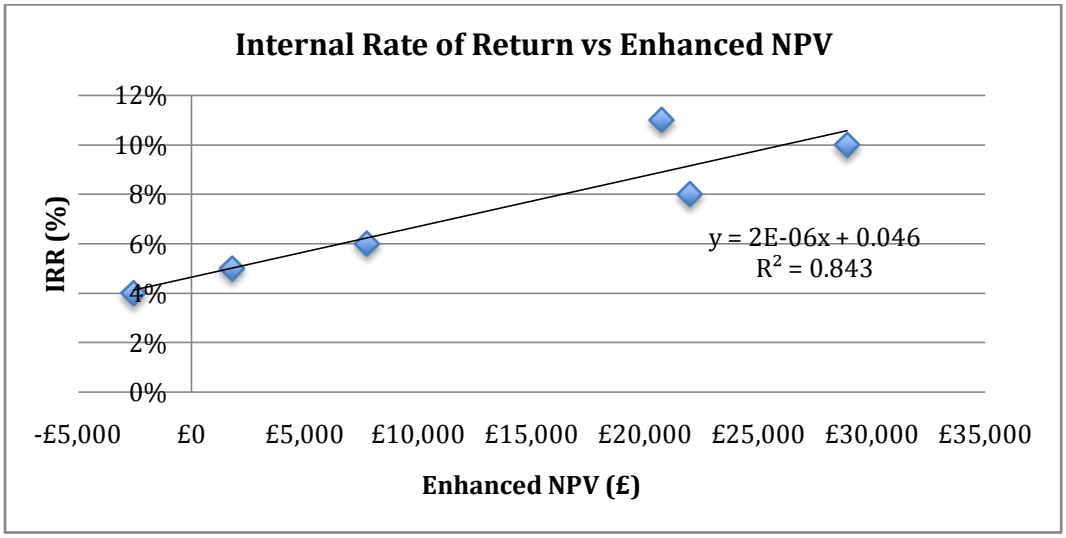


Figure 6: The internal rates of return (IRR) for each project plotted against their respective Enhanced NPVs. The resulting relationship is linear, with an $R^2$ value of 84%.

To summarise our findings, we first found that the fuzzy real options models returned values that are consistent with the traditional real options models, and that the results are robust under changing levels of fuzziness. For every project, the project valuations were increased when real options analysis was applied. Originally, some of the project NPVs were all negative, demonstrating pessimistic prospects for payback,

however the valuations according to the real options models were all positive except one (which still improved), indicating that investment prospects are improved when the values of flexibility and managerial oversight are included.

One rationale given for the use of FROV is that these models are easier to use than the traditional and well-established models in circumstances where the cash flows or strike prices are uncertain. In such cases, the process of carrying out a business case and balance sheet analysis in order to determine the cash flow and salvage value inputs for the fuzzy ROV models would yield a range of values from pessimistic to optimistic. These values would then be arranged into a triangular or trapezoidal fuzzy number and input into the relevant FROV model.

However, we argue that it is no more of an onerous process to merely run the traditional options valuation models at least twice, once for each set of pessimistic or optimistic values, in order to generate a plausible range of options prices. Furthermore, since there are many options calculators in the business world that have already implemented the traditional versions of Black-Scholes and CRR binomial trees, this would mean that practitioners could take advantage of pre-existing tools rather than having to seek out or create specialised options pricing implementations.

For example, when we examine the case of Brixton 3, where the value of the cashflow $S_0$ was £3,045, and the salvage value $X$ was £18,820, triangle and trapezoidal fuzzy numbers can be created to estimate the pessimistic and optimistic ranges for these values as follows:

$$S_{0tri}^{'} = [2890, 3045, 3198]; \;\; X_{tri}^{'} = [15000, 18820, 18920]$$
$$S_{0trap}^{'} = [2969, 3121, 80, 250]; \;\; X_{trap}^{'} = [18350, 19221, 510, 1220]$$

These fuzzy numbers were created based upon the actual cashflows, but the ranges were chosen to reflect both a reasonable level of uncertainty and/or volatility; the ranges were also selected to be deliberately non-symmetric so that the classical crisp option value would not be trivially reproduced.

The fuzzy trapezoidal numbers were input into the fuzzy Black-Scholes model, and the fuzzy triangle numbers were input into the fuzzy CRR model. The European put option price according to the fuzzy Black-Scholes model was found to be
$V_{FBS}{}' = [4147, 4337, 117, 240]$, and the put option price according to the fuzzy CRR model was found to be $V_{FCRR}{}' = [4186, 4214, 4209]$. For comparison, the crisp Black-Scholes value was $V_{BS} = 4262$, and the crisp CRR put value was $V_{CRR} = 4197$.

The fuzzy put values yield an option range of about 4030 to 4577. By comparison, the traditional crisp Black-Scholes model was then used for the following two pairs of inputs: $S_{01} = 3121$, $X_1 = 18350$; and $S_{02} = 2969$, $X_2 = 19221$. These values represent the optimistic cashflow values paired with the pessimistic salvage values, and vice versa, to obtain the indicative option price spread. The Black-Scholes put values resulting from these two sets of values are $V_1 = 4045$ and $V_2 = 4441$, with an average put value of 4243, which is within 2% of $V_{BS}$ and $V_{CRR}$ given above. This demonstrates that rather than having to deal with the complexity of implementing a new fuzzy ROV model, comparable values can be obtained through the use of traditional real options valuation by using the same pessimistic and optimistic values for cashflow and salvage.

## 6. Conclusions

The main objective of the analysis here has been to explore whether fuzzy real options could improve the investment prospects ofrenewable energy and energy efficiency projects. Four real options analysis methods were used to value the salvage option present in three urban rooftop solar projects. Two of these methods, Fuzzy Black-Scholes and Fuzzy Binomial, used fuzzy numbers for analysis. These techniques take fuzzy cash flows and salvages as inputs, and output the options prices as fuzzy numbers. The values produced from the fuzzy options modelswere found to be consistent with the outputs from the traditional options models. This consistency in values demonstrates that, despite allowing for uncertainties in the inputs, reliable outputs can be attained from both of these fuzzy option valuation methods. Furthermore, the use of real options valuation to adjust project NPVs retains a linear relationship with IRR, confirming that both can be useful as project viability indicators.

For the projects analysed in our case study, their financial valuations were indeed improved by real options analysis, and fuzzy real options models gave robust and consistent results. Ideally, the flexibility inherent within fuzzy real options valuation would help with incorporating the policy and cashflow uncertainties that are common to many renewable energy projects, however we found that similar results could be found from

traditional ROV with less effort.Therefore, in relation to the effective applicability for practitioners of the FROV models, we conclude that they require some work in implementation, since standard options calculators cannot be used. Similar results (i.e., a set of option values) can be found by merely running the traditional ROV models over a range of crisp inputs. The resulting range of crisp ROV option values is very close to those returned by the FROV models, which brings into question the usefulness of FROV techniques, particularly in the scope of renewable energy investment.

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